

Learning Where to Look: Reliable Certificates Under Scarce Ground Truth

Gautam Dasarathy

Arizona State University | Amazon

<http://gautamdasarathy.com>

CNI Seminar, IISc
November 2025

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Or

Label-efficient Two-Sample Testing

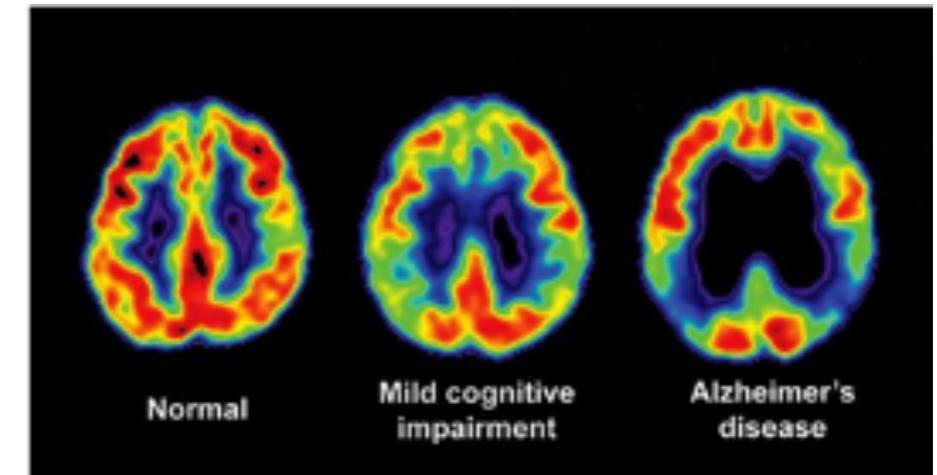
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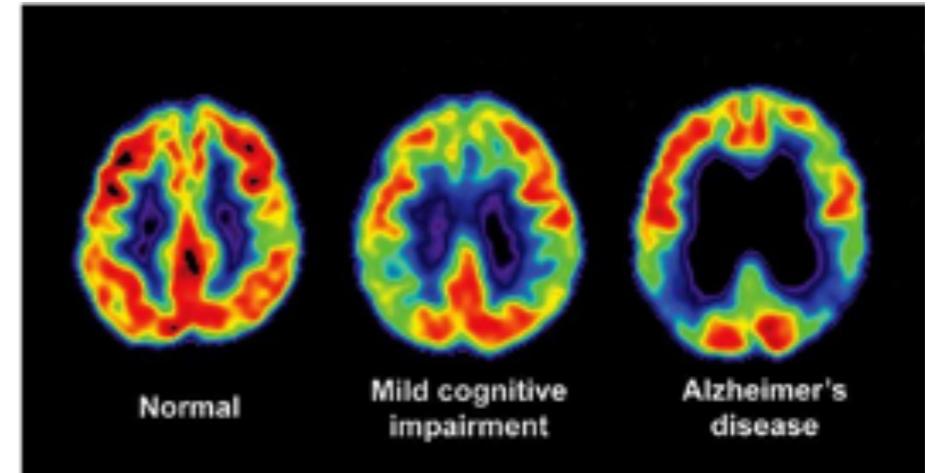
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Can Digital Tests Stand in for PET Scans?

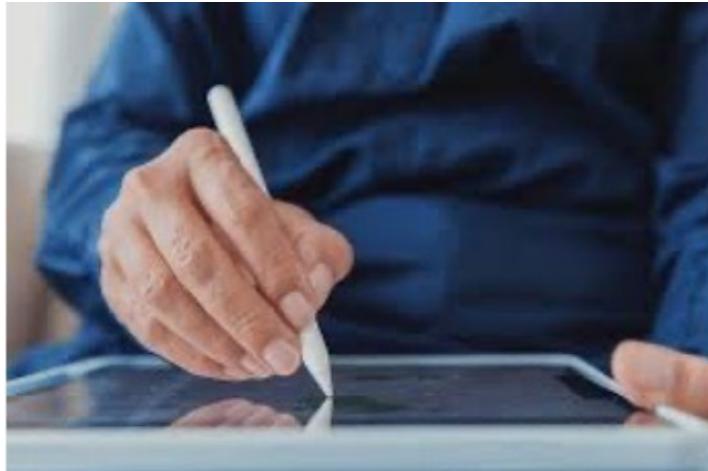


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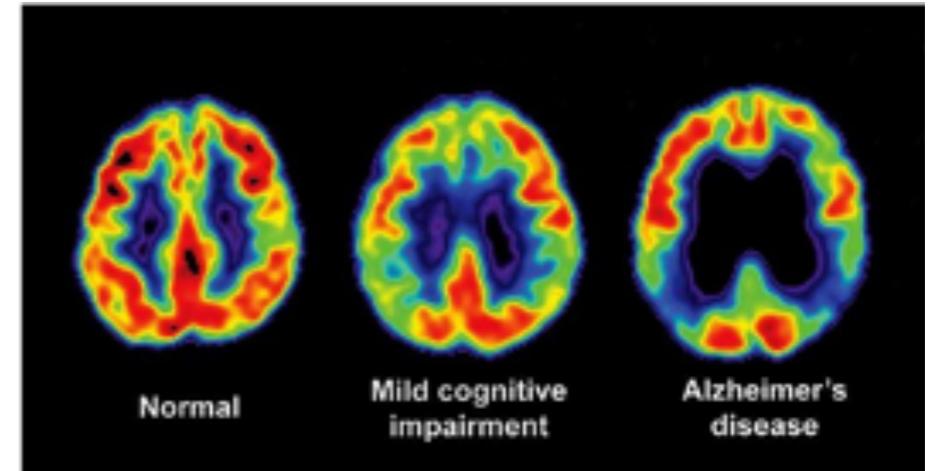


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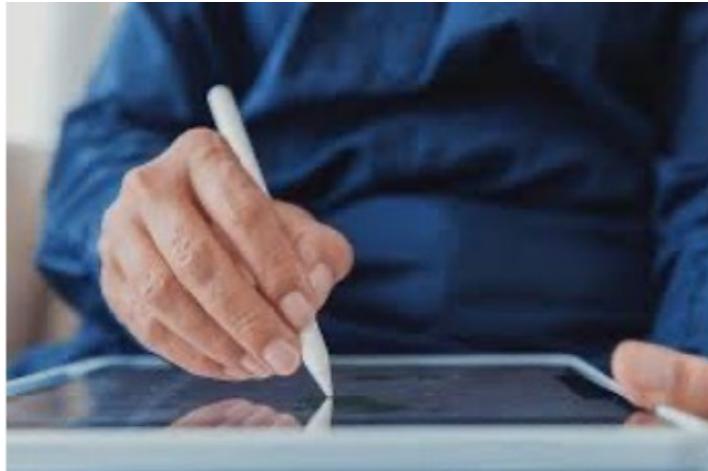


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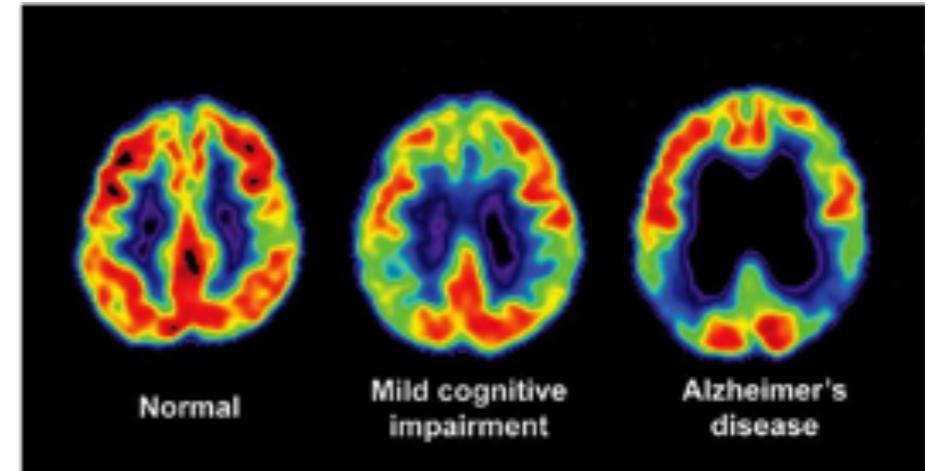
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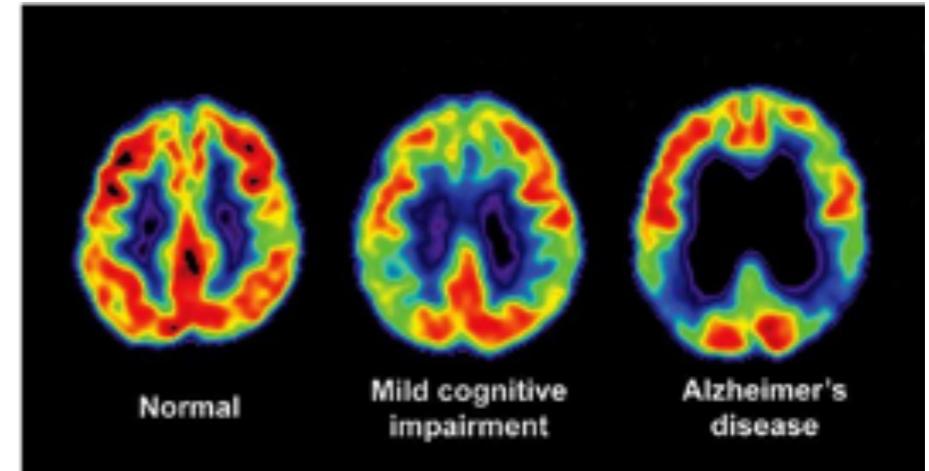
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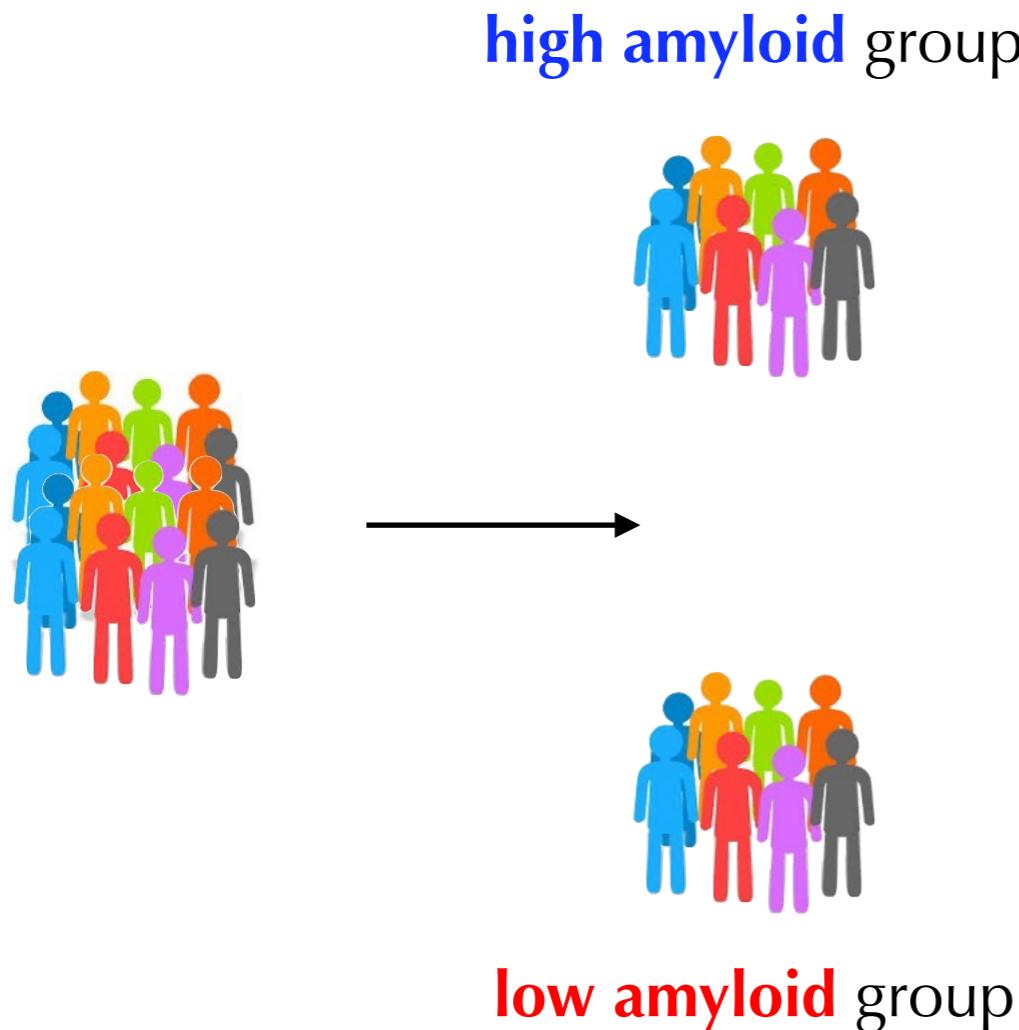
Is this any good?
Do the **digital test distributions differ** between high- and low-amyloid groups?

Can Digital Tests Stand in for PET Scans?

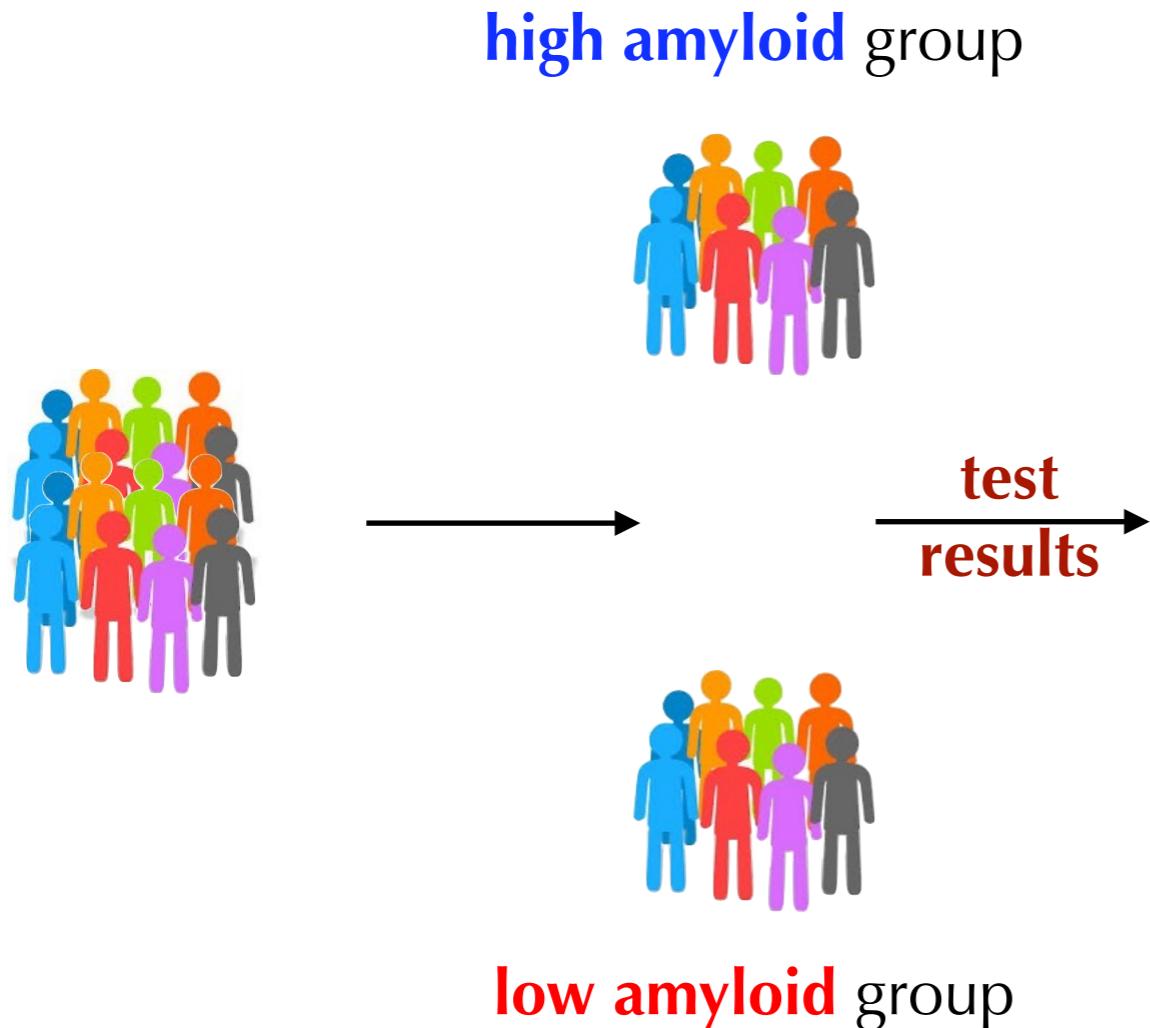
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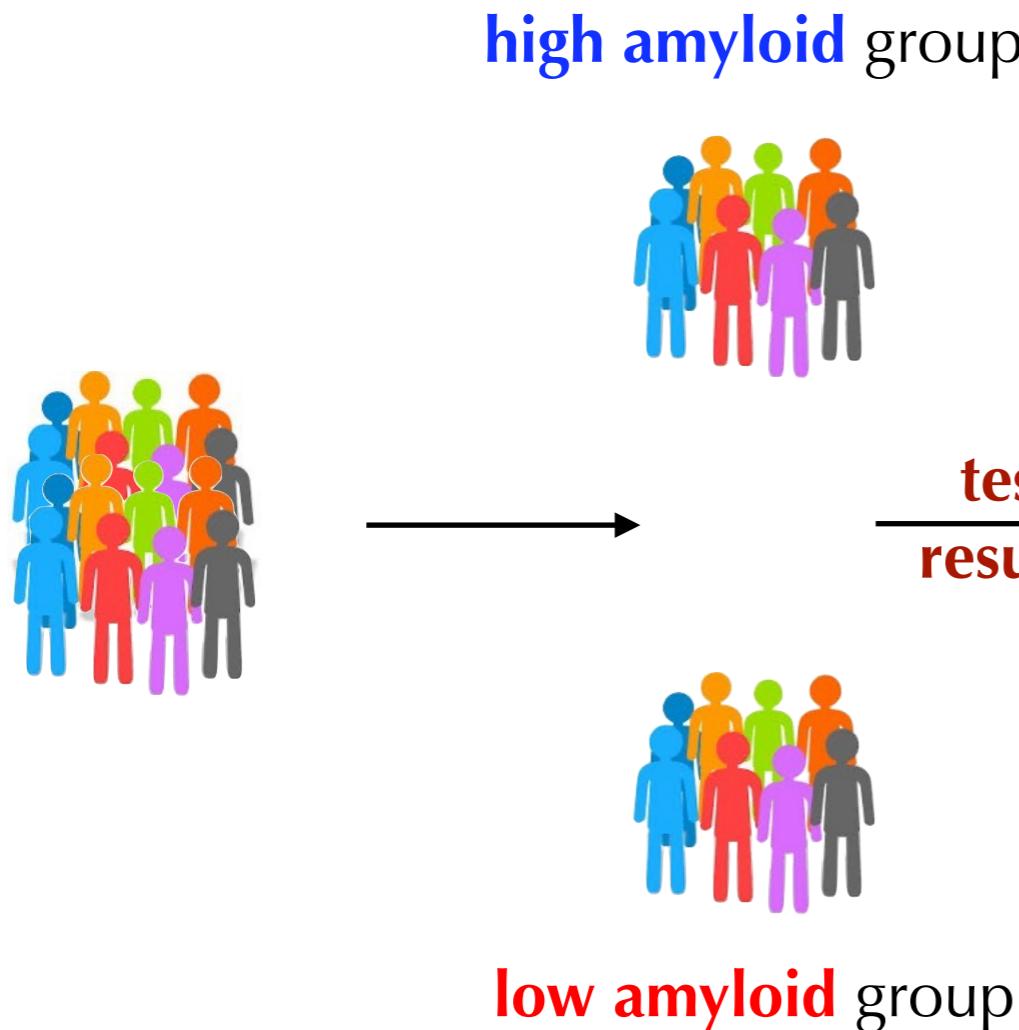
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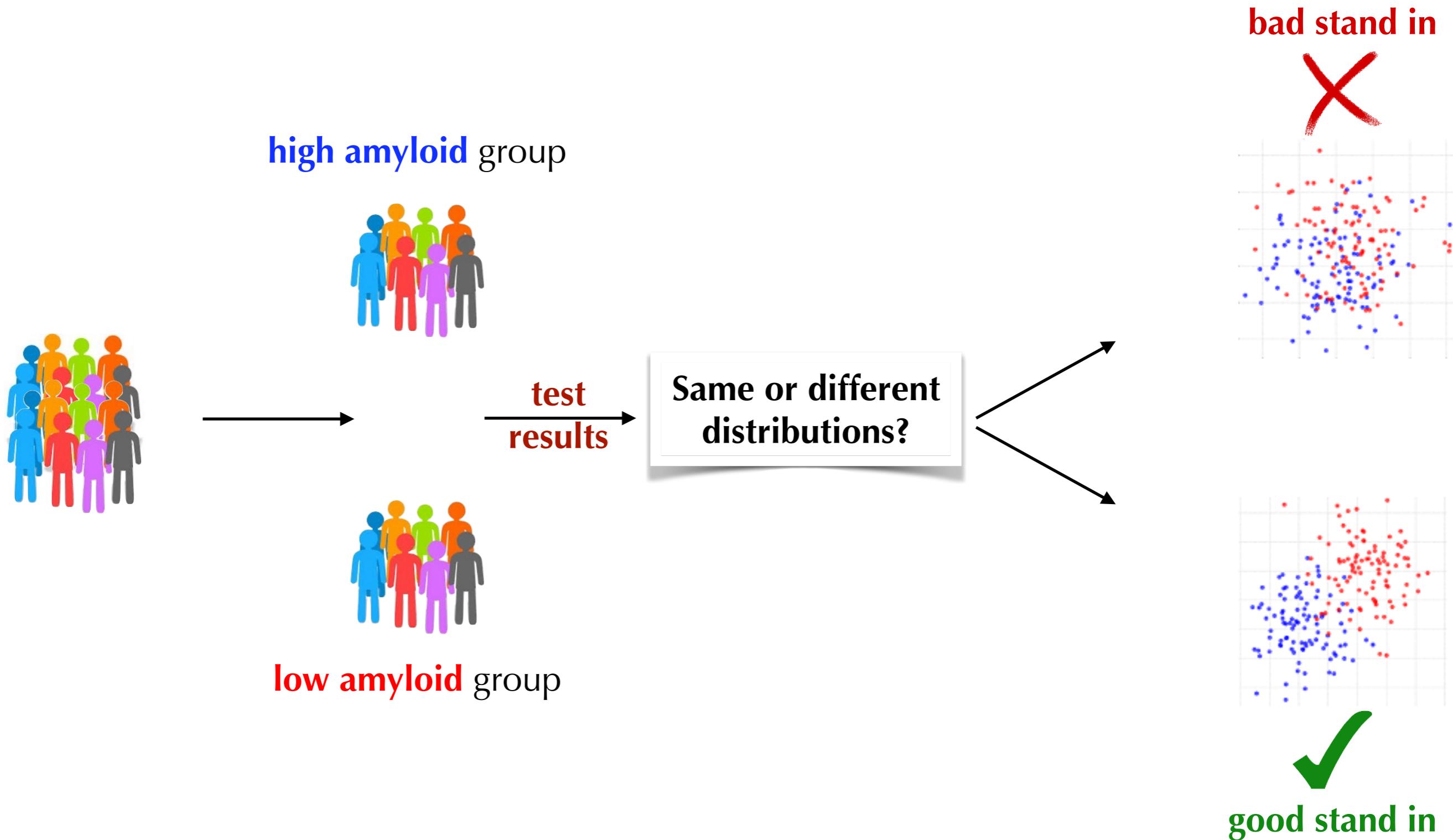
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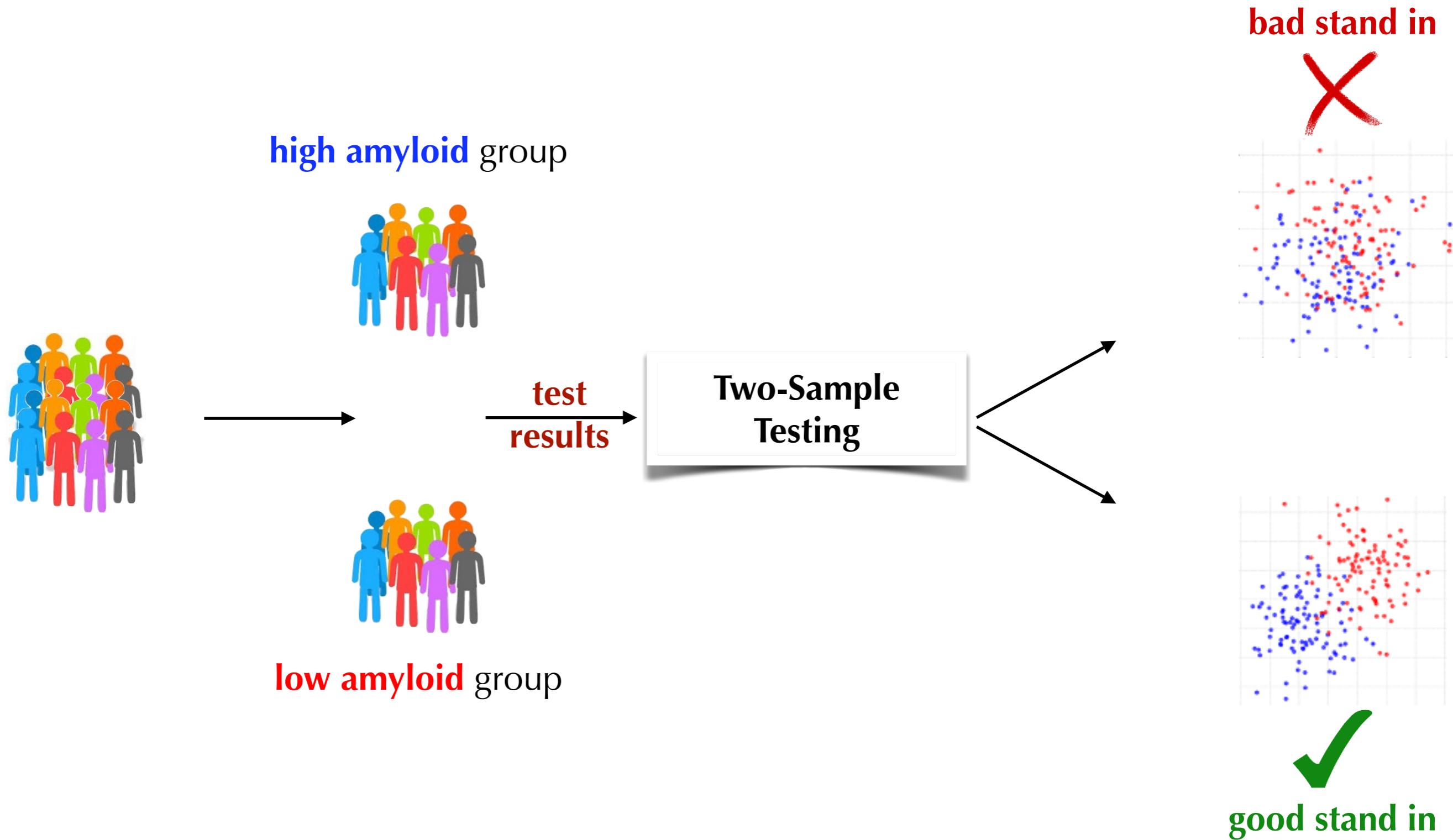
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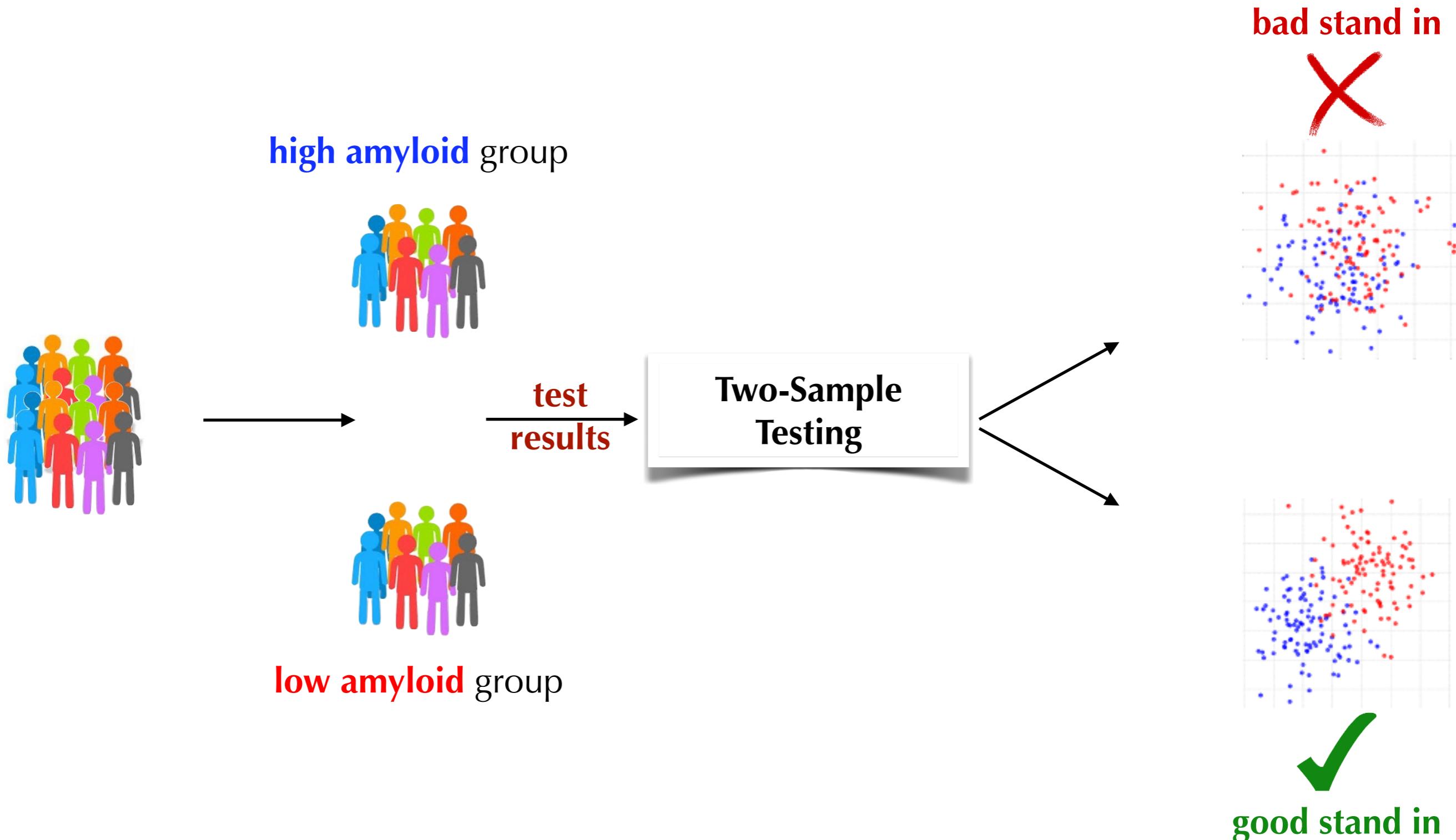
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Can Digital Tests Stand in for PET Scans?



Two-sample testing: Given samples $X_1, \dots, X_m \sim P$ (high-amyloid) and $Y_1, \dots, Y_n \sim Q$ (low-amyloid), Test: $H_0 : P = Q$ vs $H_1 : P \neq Q$

Two Sample Testing is Everywhere

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**Digital health
sensor validation**

Two Sample Testing is Everywhere



**Digital health
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**Fraud intervention
efficacy**

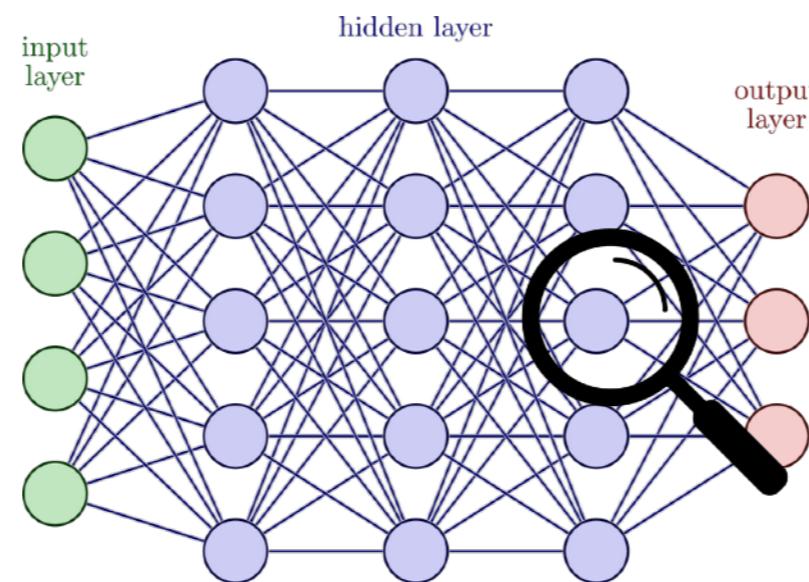
Two Sample Testing is Everywhere



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Model Monitoring / ML OPs:
data drift relative to training?

2ST: More than a Century of Data-Driven Science



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Gosset (1908)

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Wald-Wolfowitz (1940s)

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2ST: More than a Century of Data-Driven Science



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Modern era



Wald-Wolfowitz (1940s)



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Setting Up The Two Sample Testing Problem

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Given: $X_1, \dots, X_m \sim P$ and $Y_1, \dots, Y_n \sim Q$ (iid). Perform the following **hypothesis test**

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We usually compute a **statistic from the data**, and reject H_0 if the value is *too extreme*

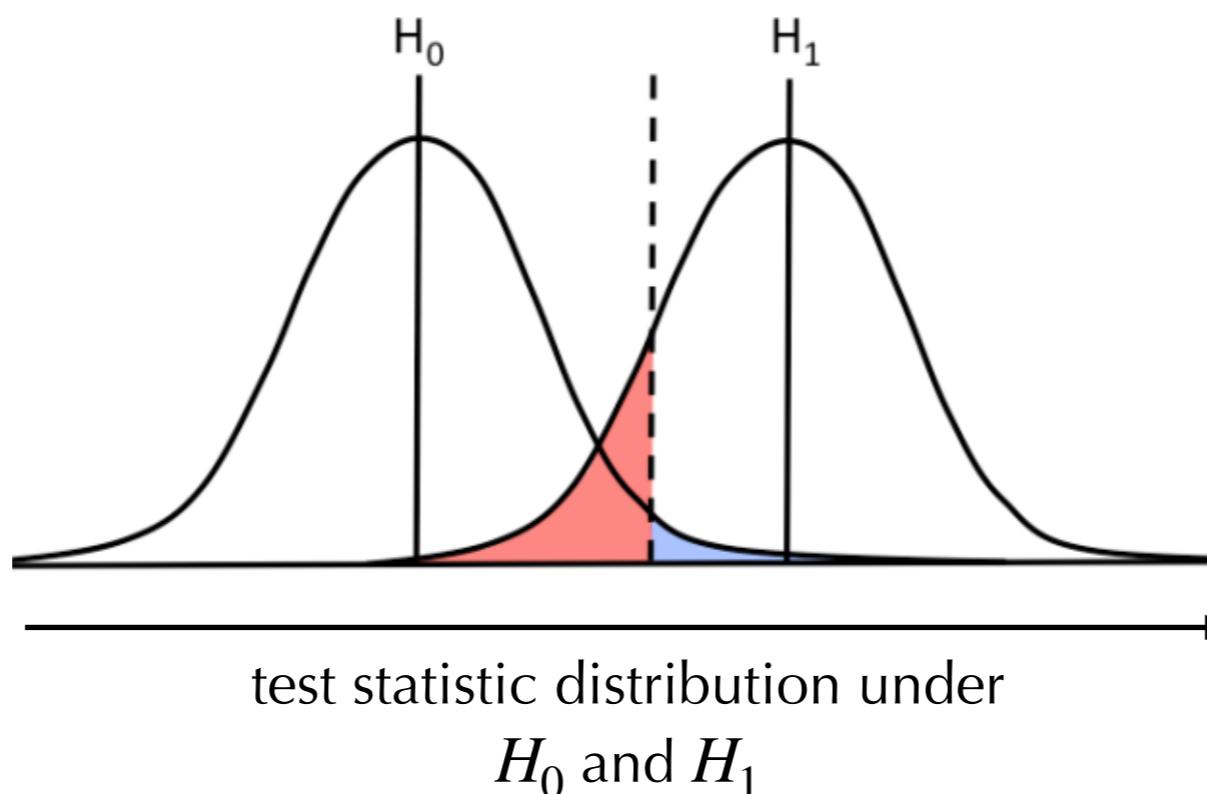
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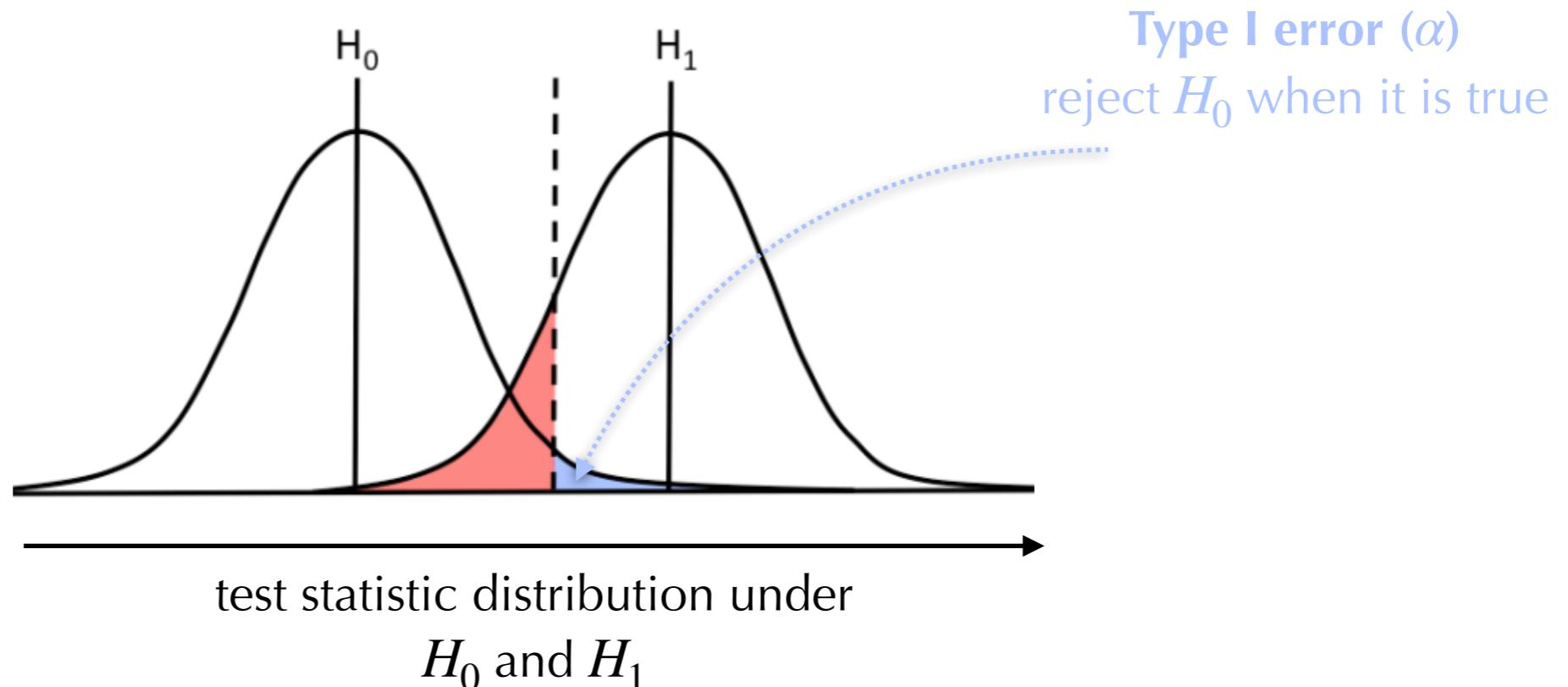
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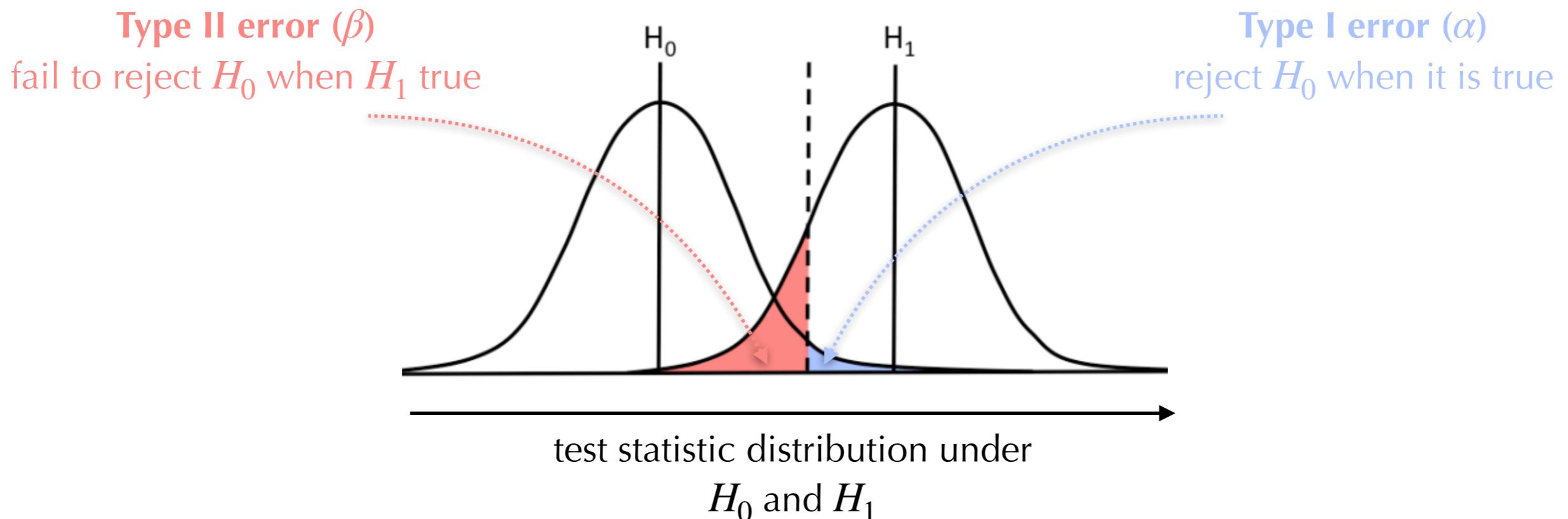
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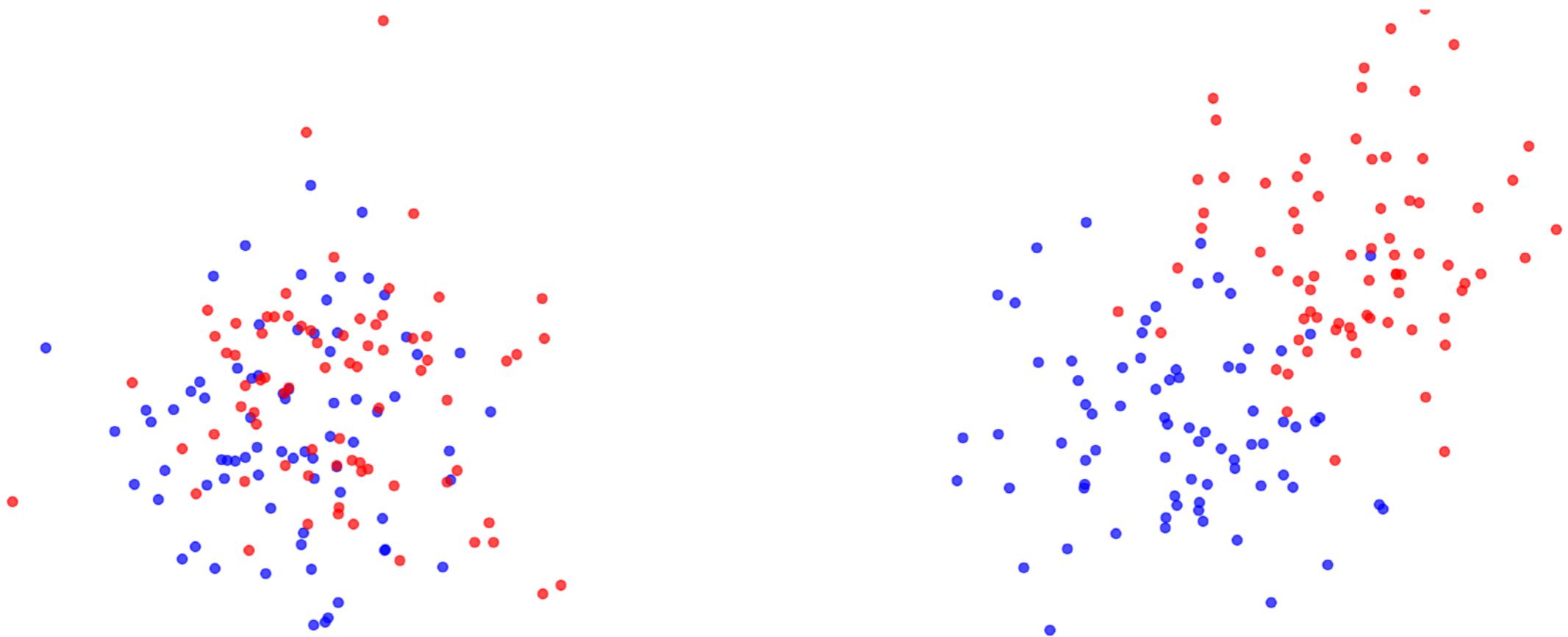
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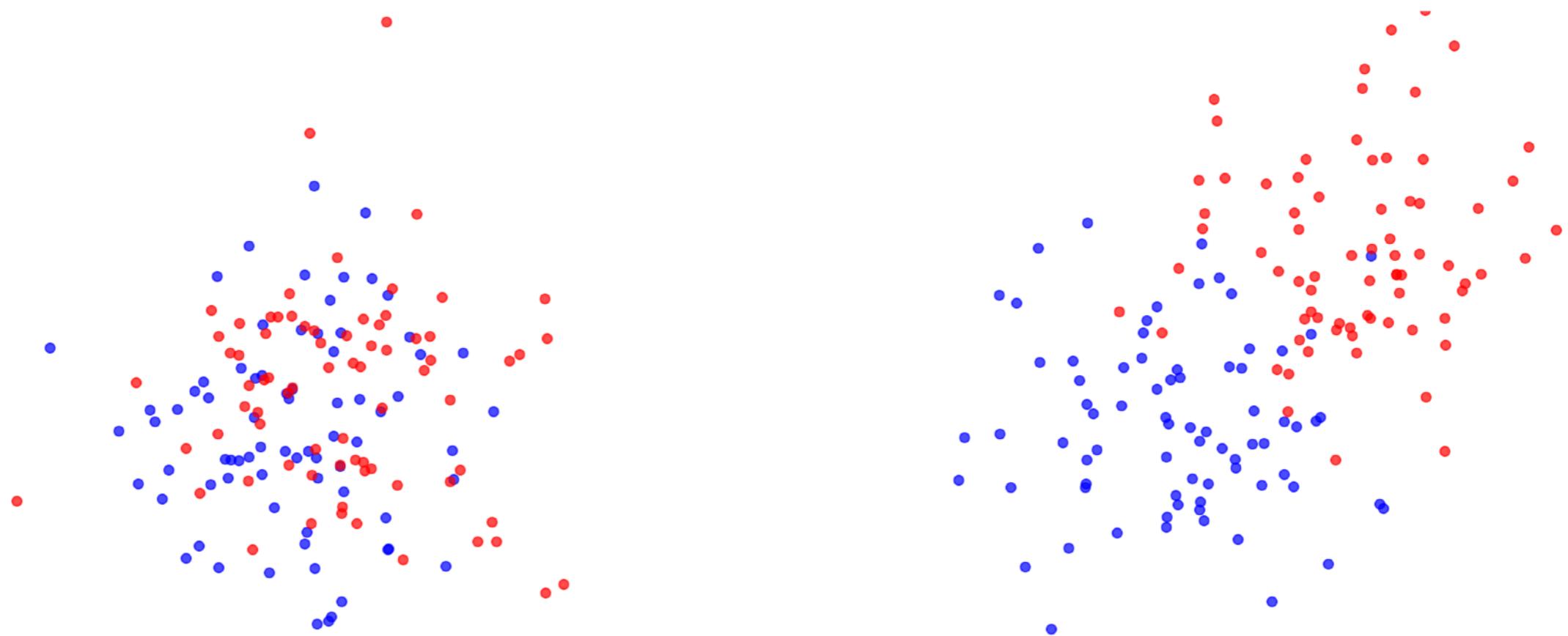
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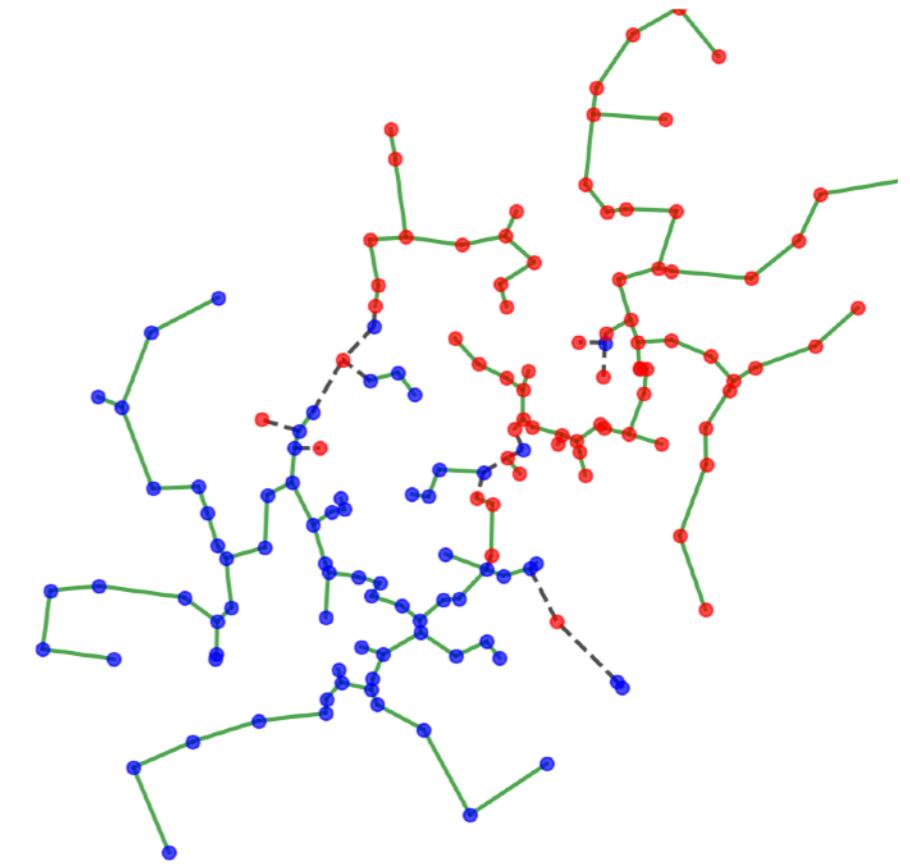
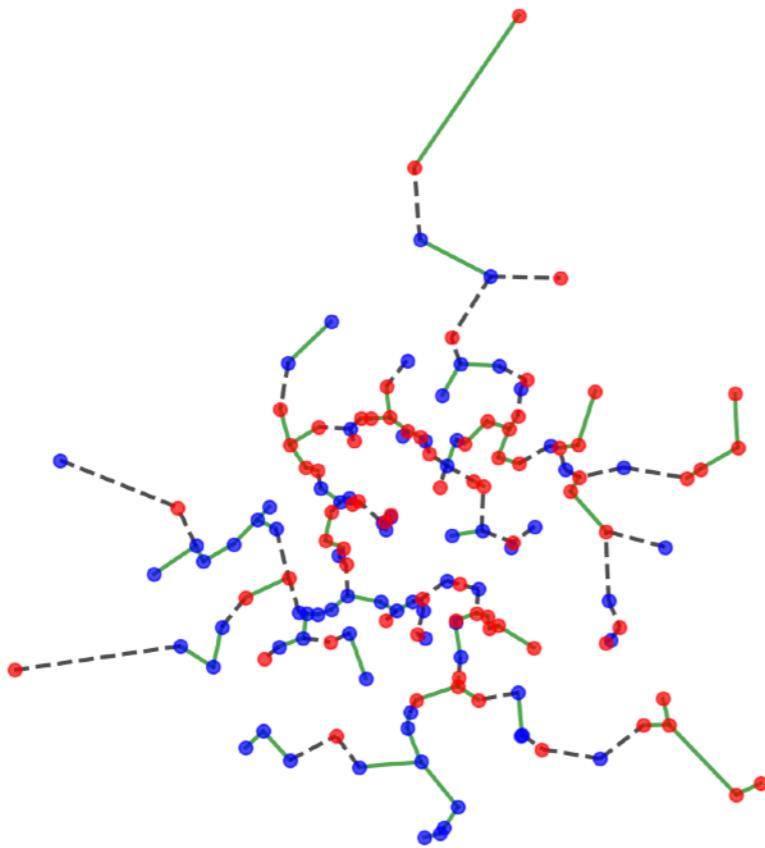
Classical Two Sample Test in Action: Friedman-Rafsky



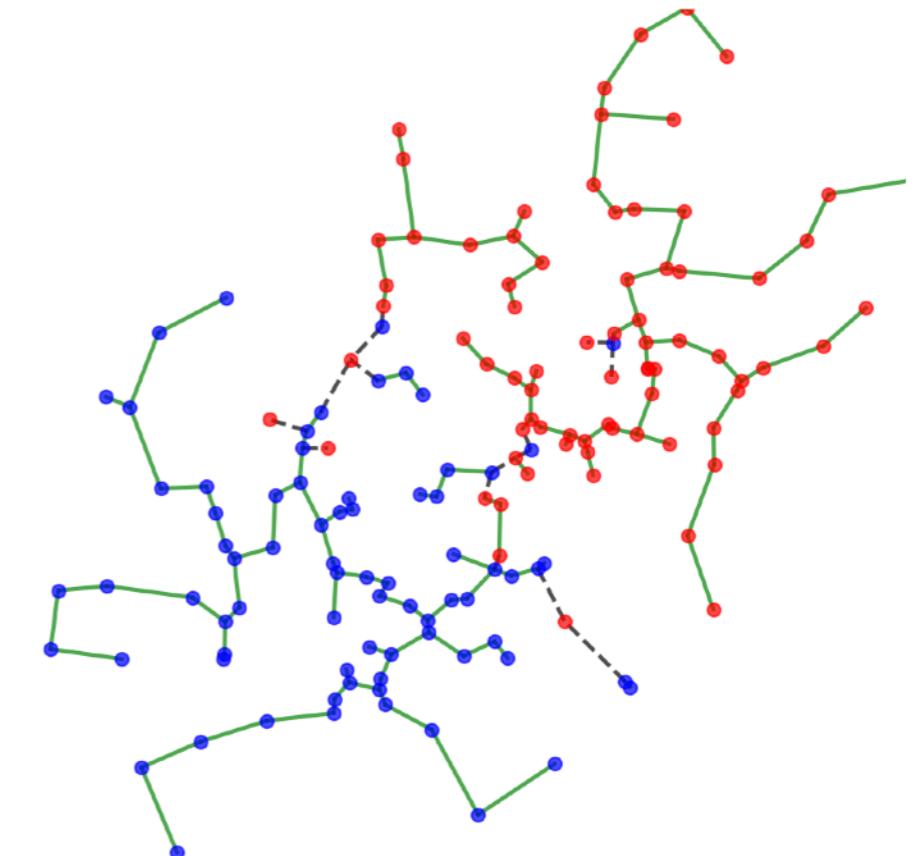
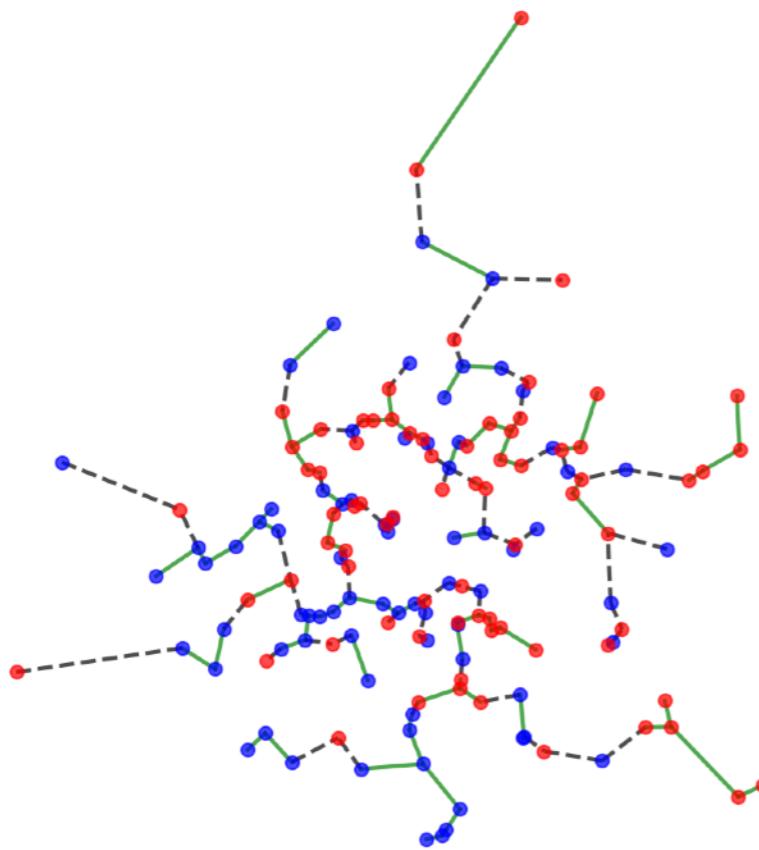
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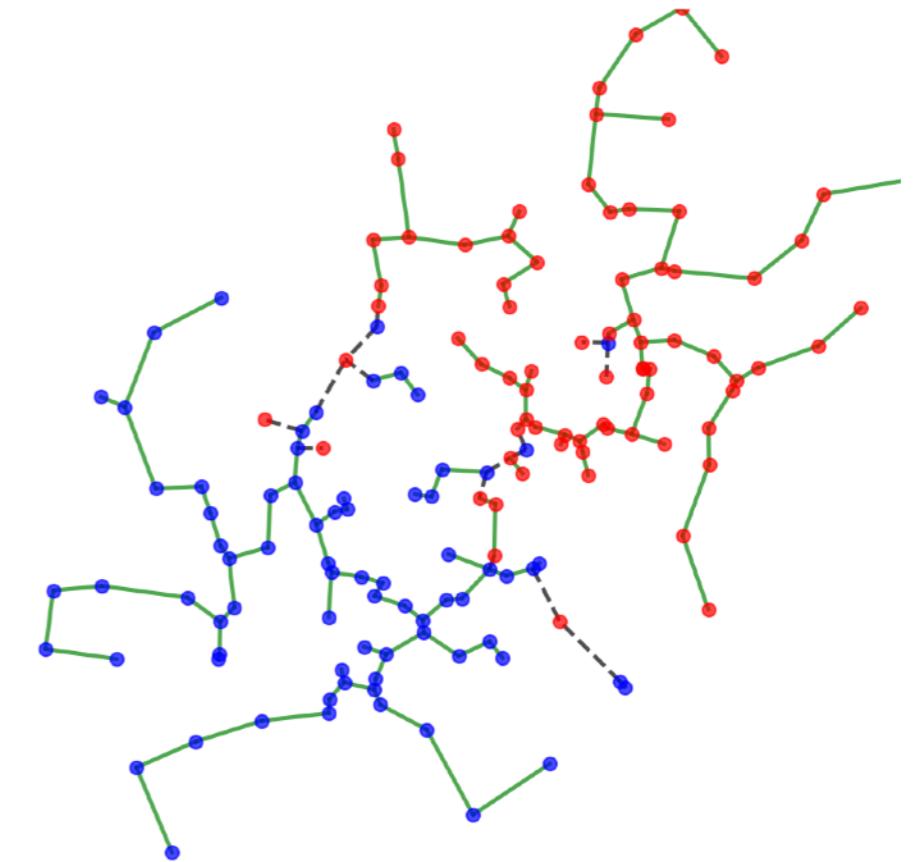
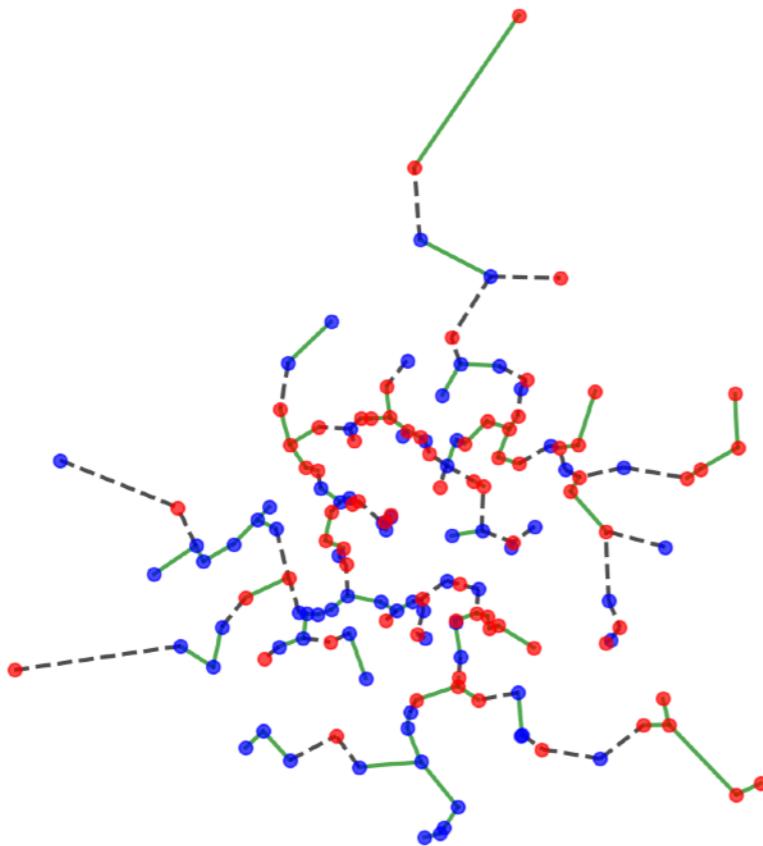
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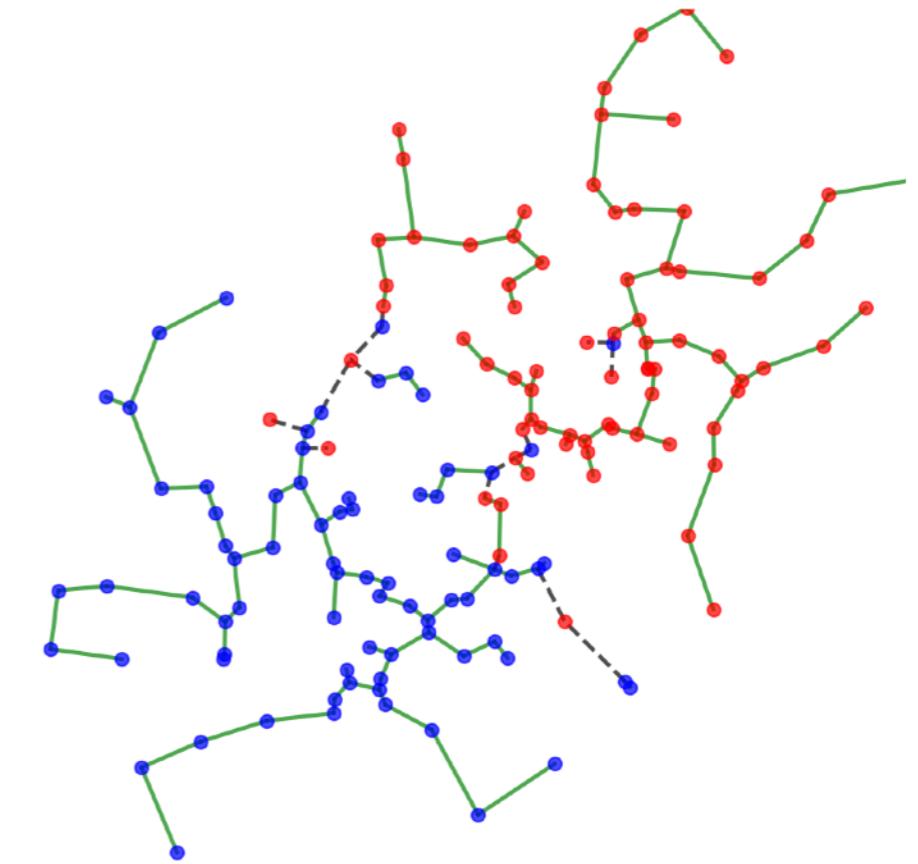
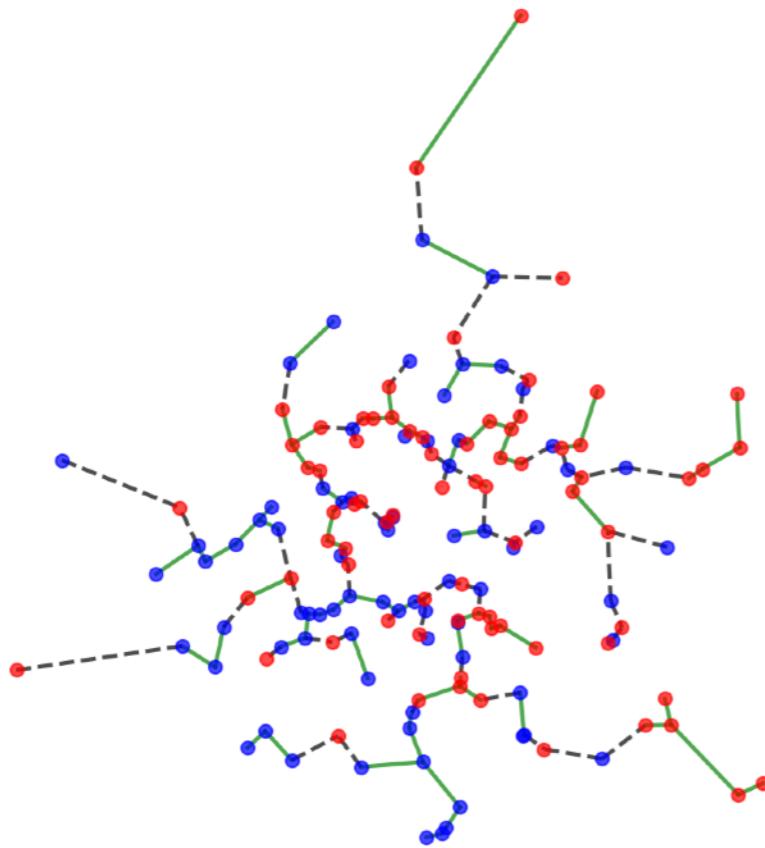
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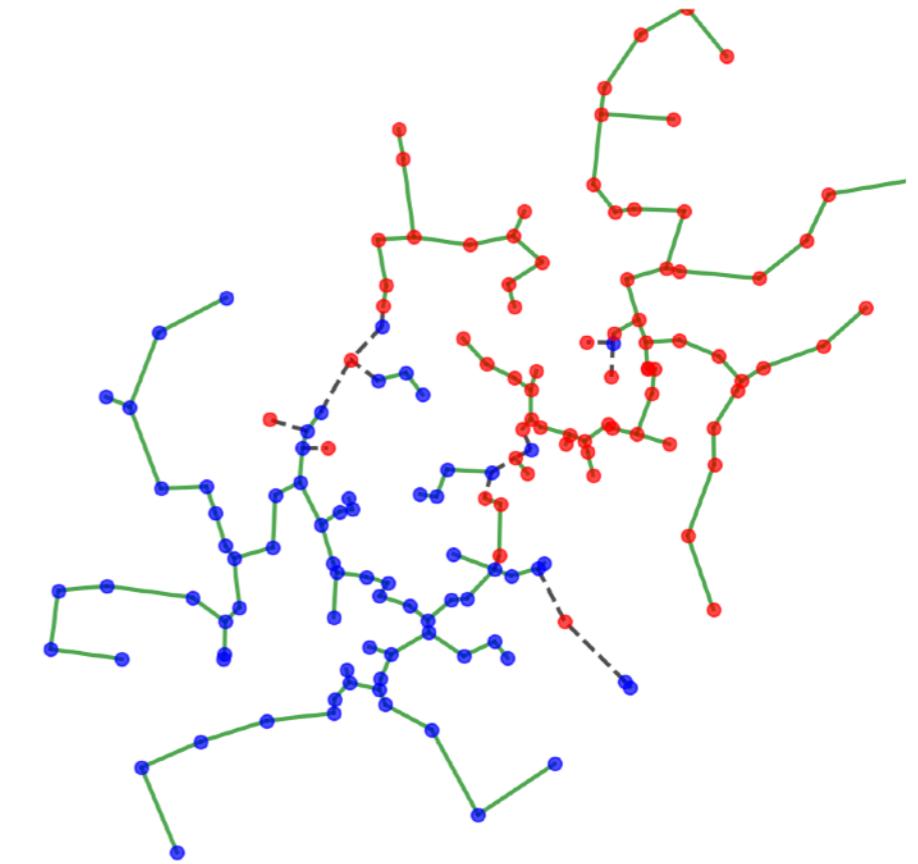
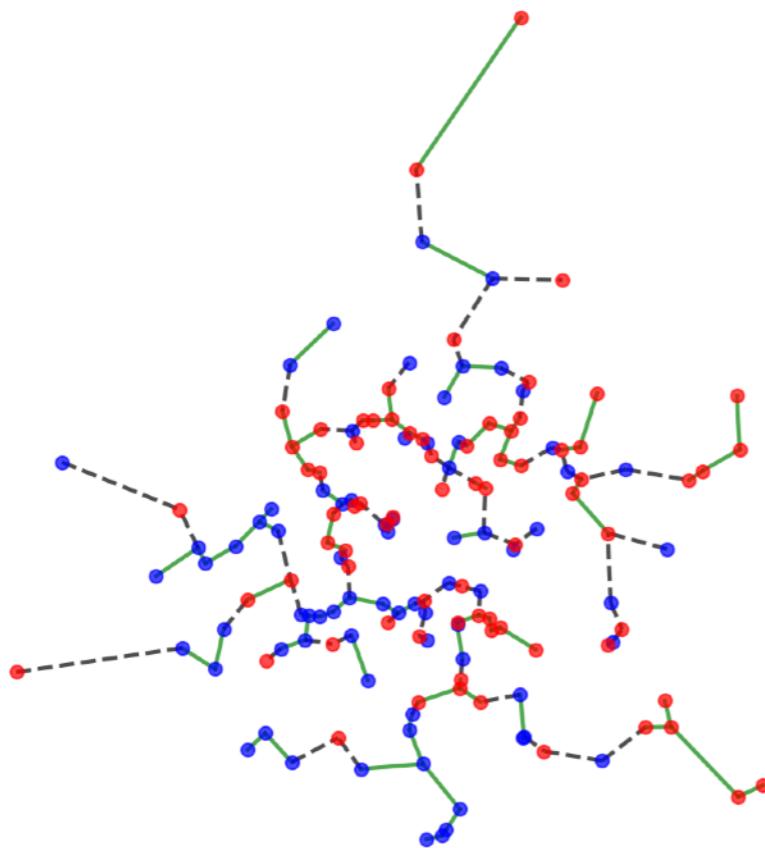
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Friedman, Jerome H., and Lawrence C. Rafsky. "Multivariate generalizations of the Wald-Wolfowitz and Smirnov two-sample tests." The Annals of Statistics (1979): 697-717.

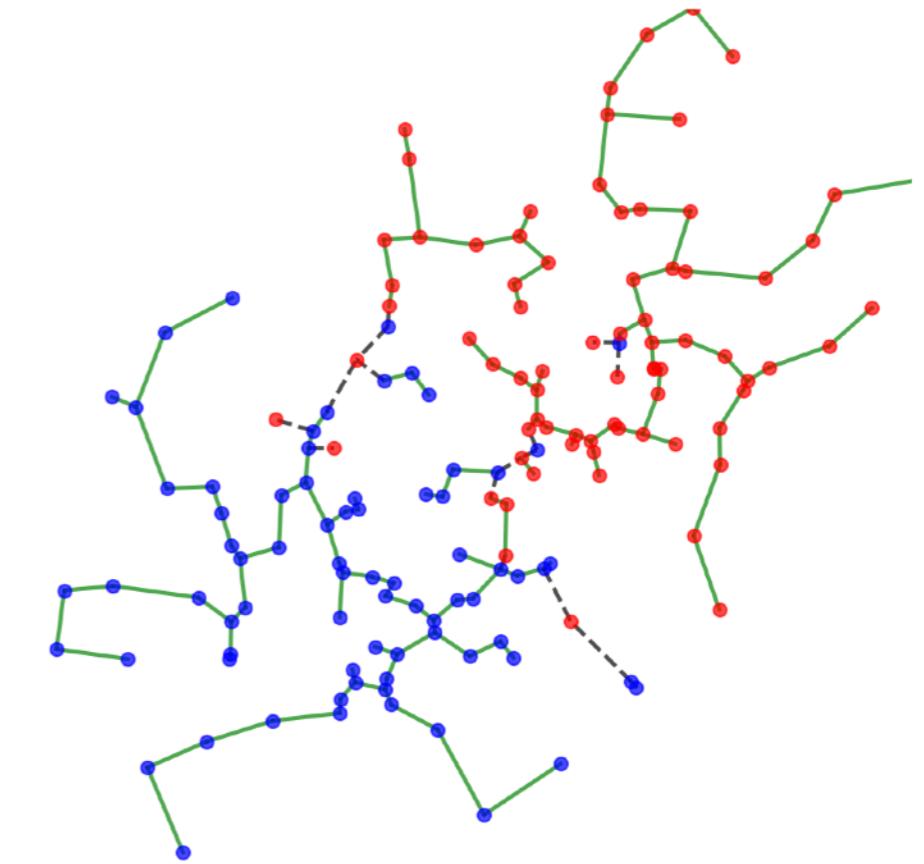
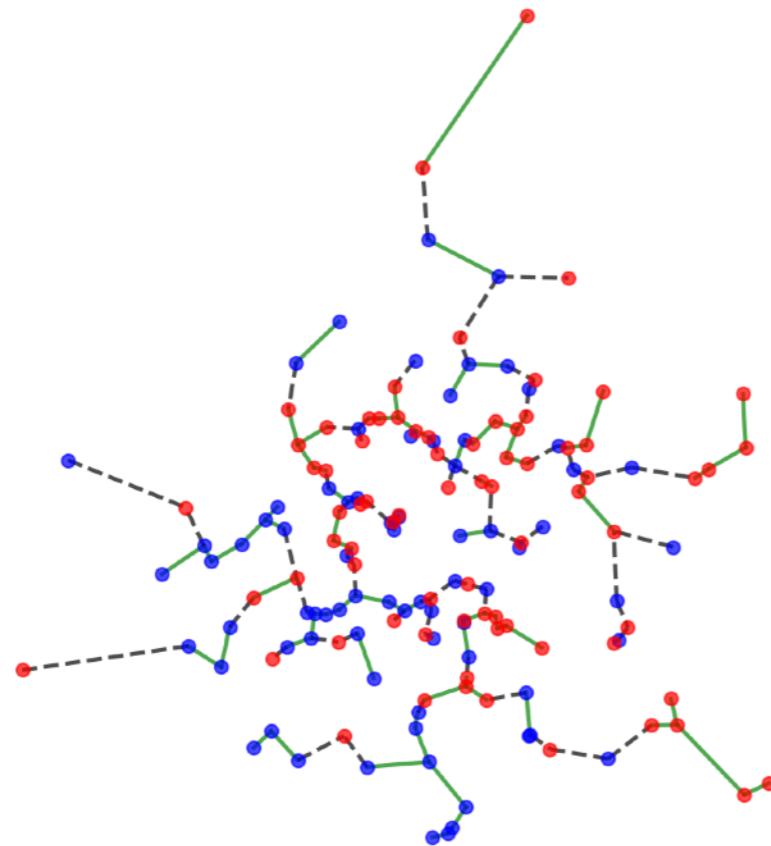
Henze, Norbert, and Mathew D. Penrose. "On the multivariate runs test." Annals of statistics (1999): 290-298.

Classical Two Sample Test in Action: Friedman-Rafsky



Theorem 1 (FR '79) The normalized cut-edge count R is **asymptotically normal** under H_0 ; it's mean and variance have analytical expressions —> can construct a **permutation test**

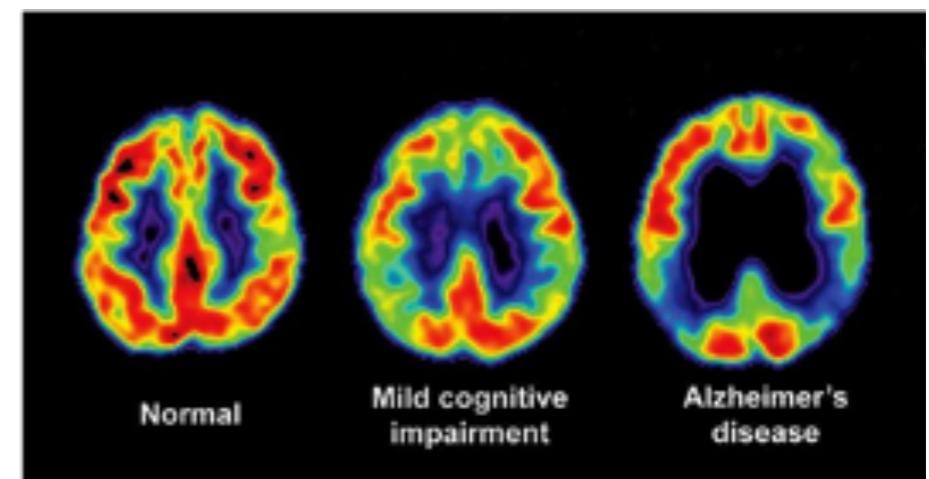
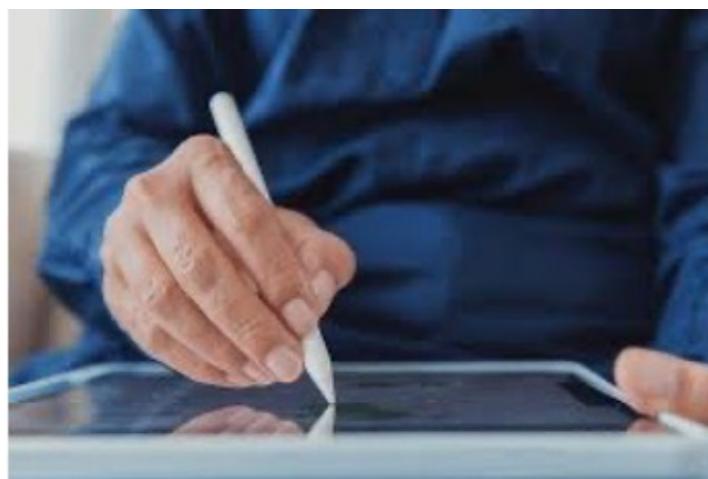
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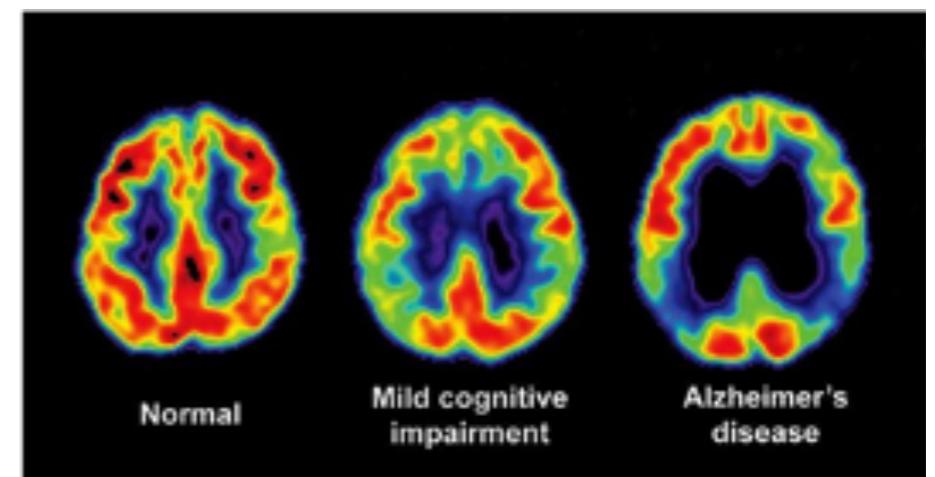
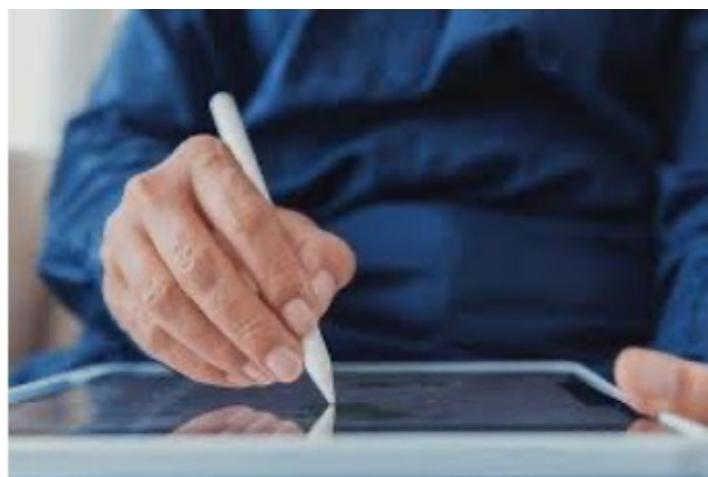
Theorem 1 (FR '79) The normalized cut-edge count R is **asymptotically normal** under H_0 ; it's mean and variance have analytical expressions —> can construct a **permutation test**

Theorem 2 (HP '99) The FR test is **consistent**. In particular $R/m + n \rightarrow c \left[1 - D_f(P\|Q) \right]$

The Catch: Group Memberships are Often Expensive to Determine

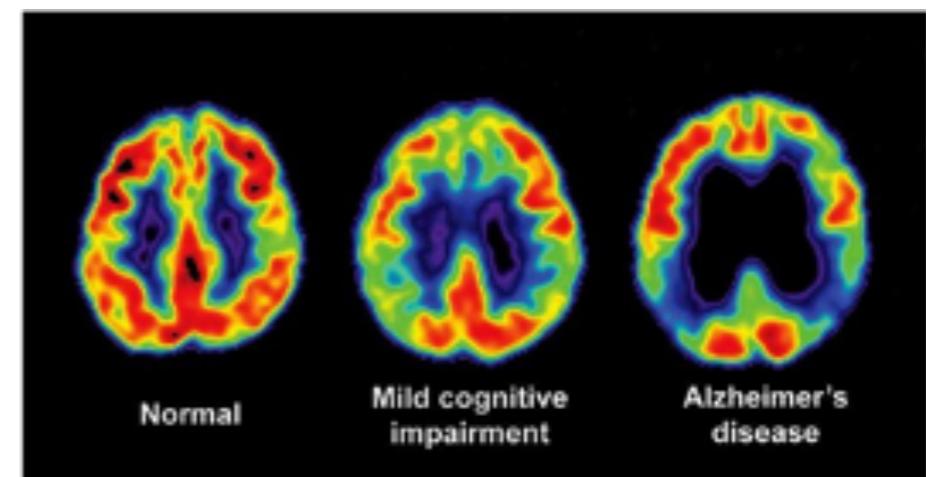
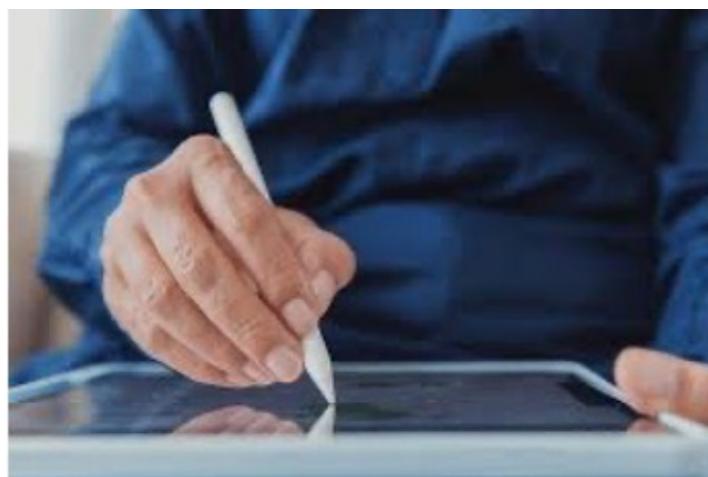


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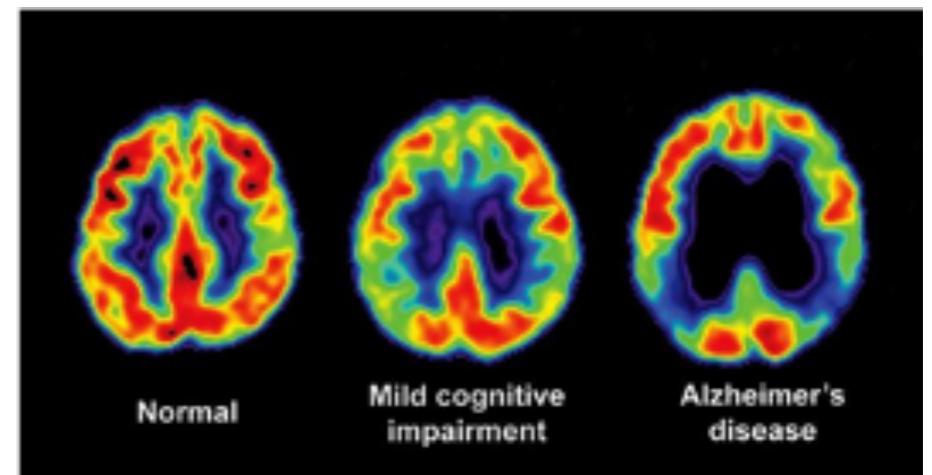


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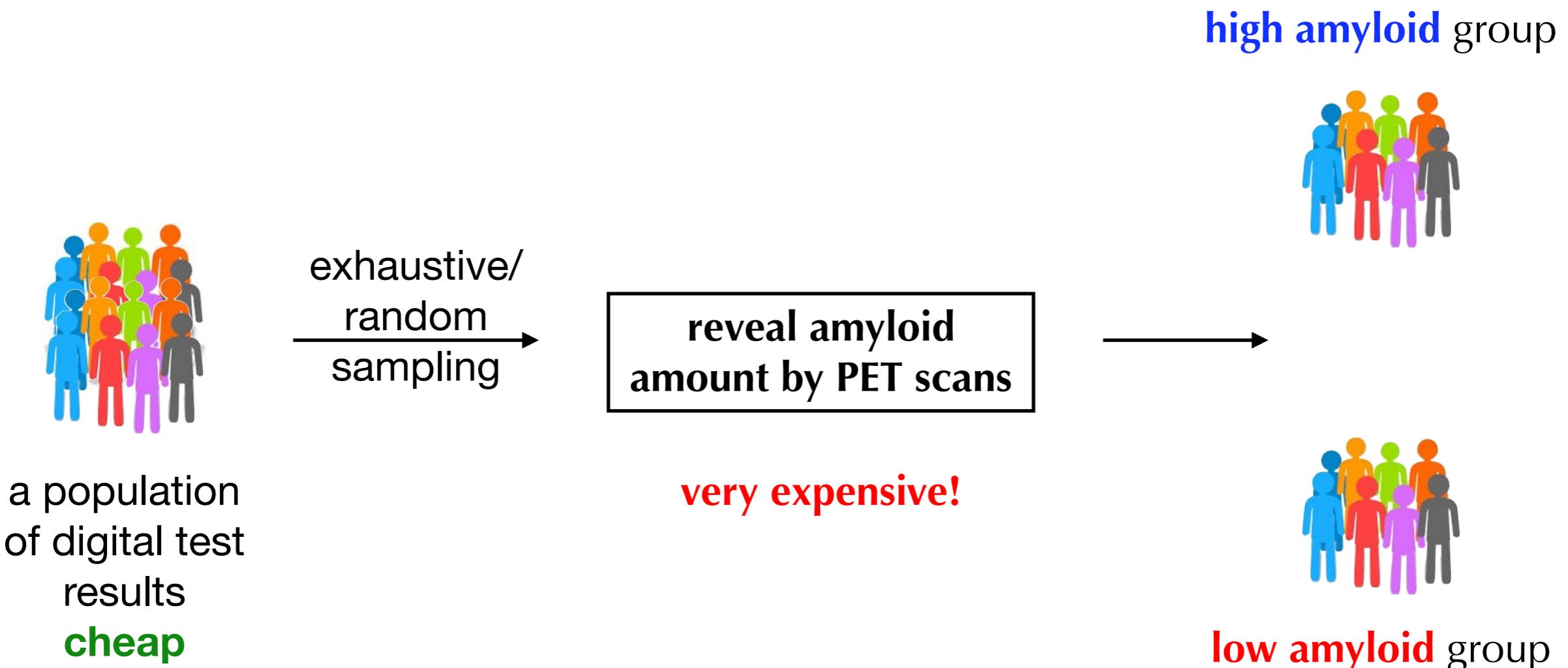
Back to our example: validating digital biomarkers for AD



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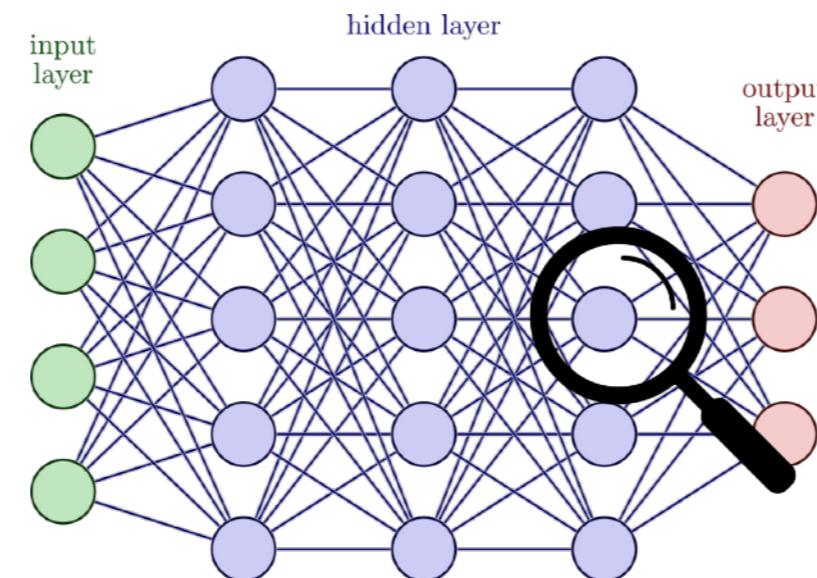
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Digital health sensor validation
digital health data easy;
lab tests expensive



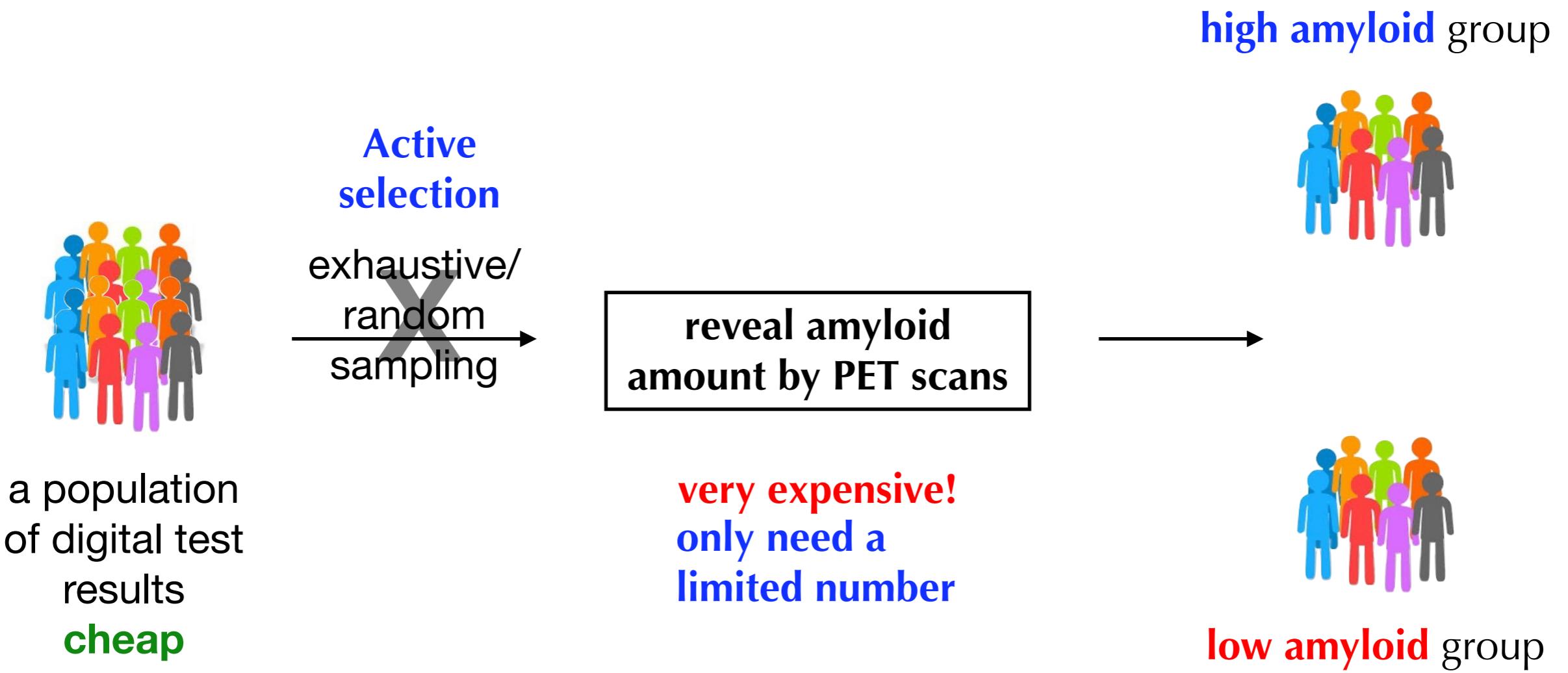
Financial Fraud Detection
transactions features easy to obtain;
classifying is expensive



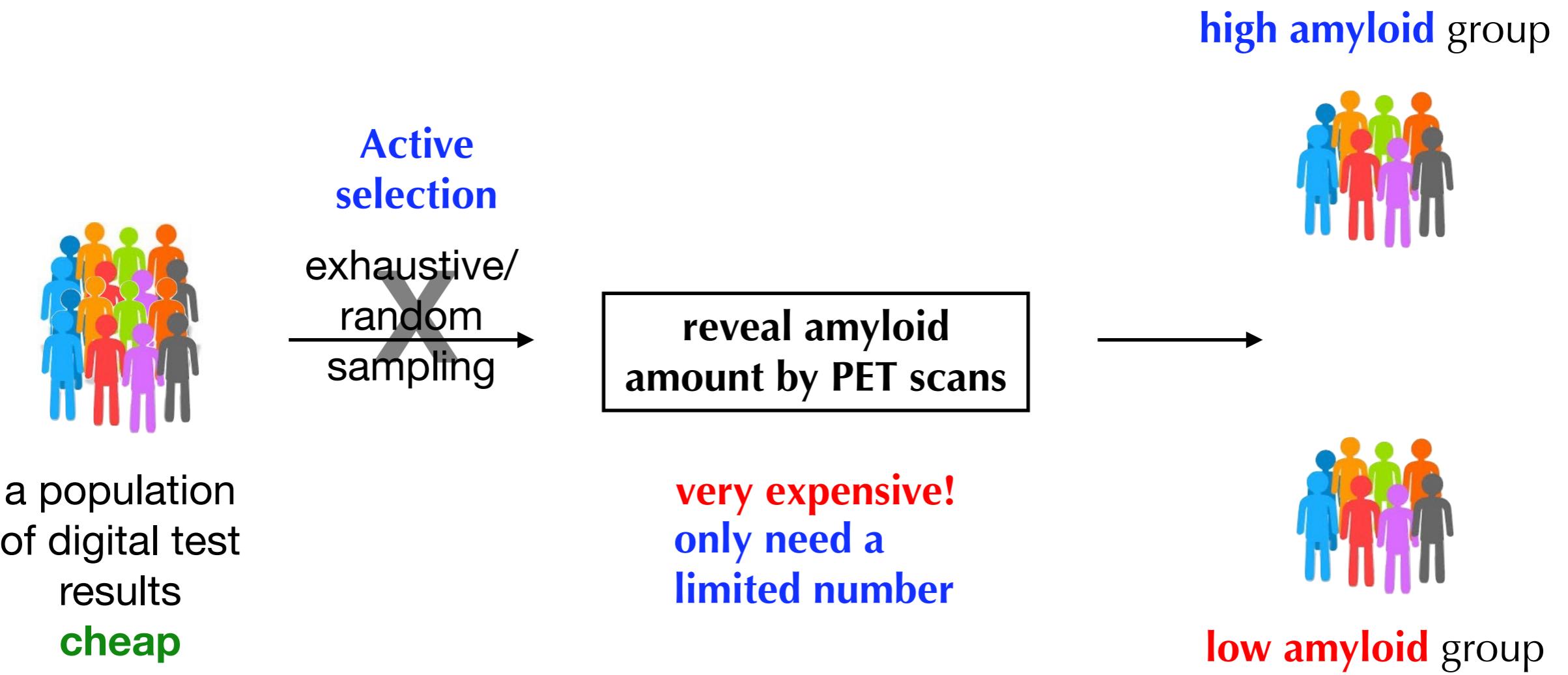
Model Monitoring / ML OPs:
data drift relative to training?

model outputs/perf easy;
post-deploy groundtruth hard

Active Querying for Two-Sample Testing

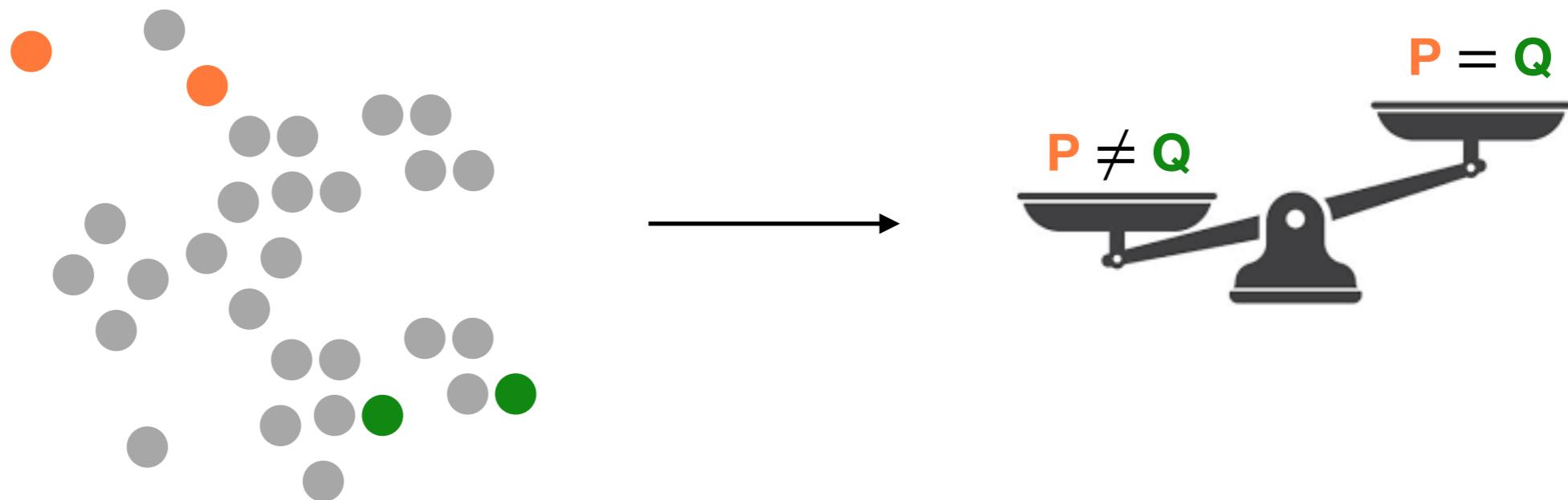


Active Querying for Two-Sample Testing



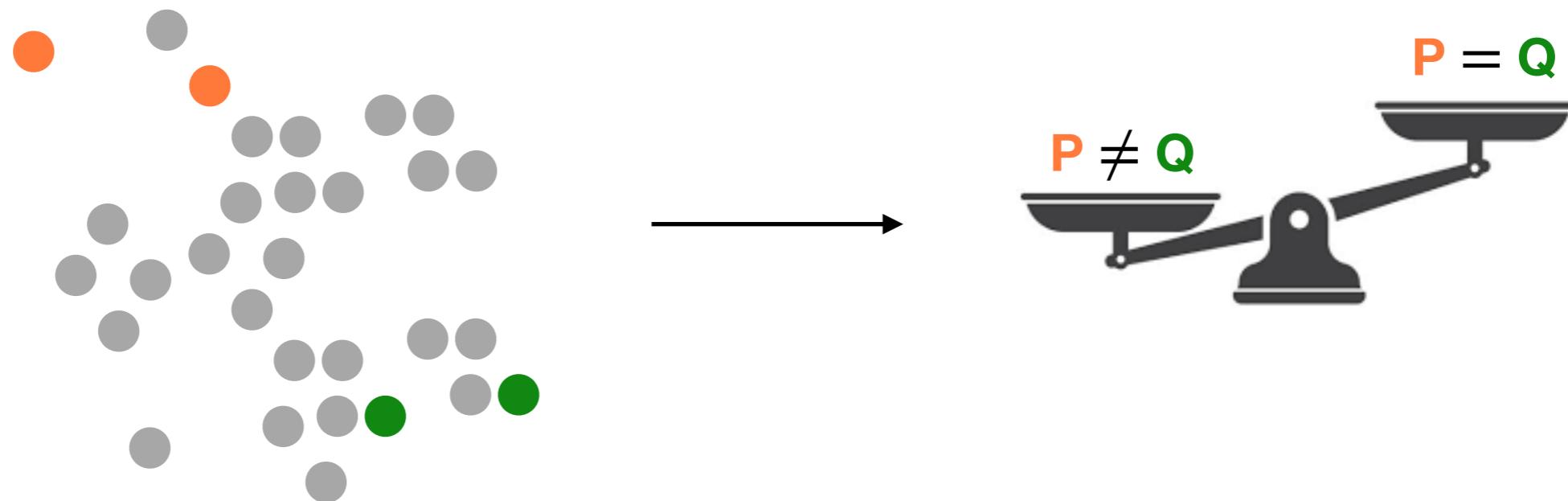
Idea: Carefully (and adaptively) select digital test results (features) and query their group memberships (i.e., PET scans)

A New Two-Sample Testing Problem



A New Two-Sample Testing Problem

Given a **large population of sample features** and a **limited labeling (group-membership ascertaining) budget**, our goal is to develop a **label-efficient two-sample test** to determine **whether the two samples are drawn from the same or different distributions**.



A Different Perspective on Two-Sample Testing

Data Model

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Data Model

$$Z \sim \text{Ber}(\theta)$$

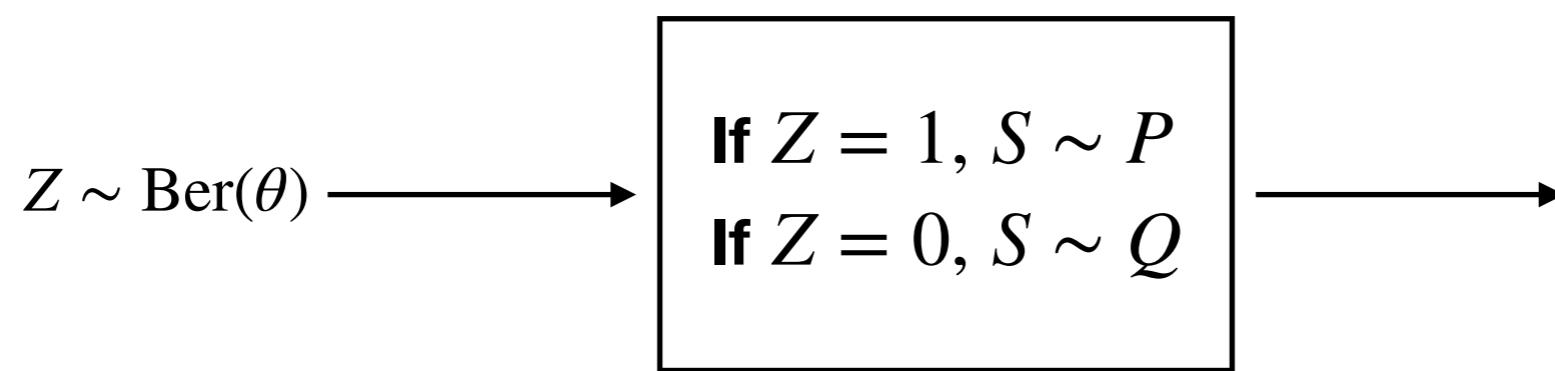
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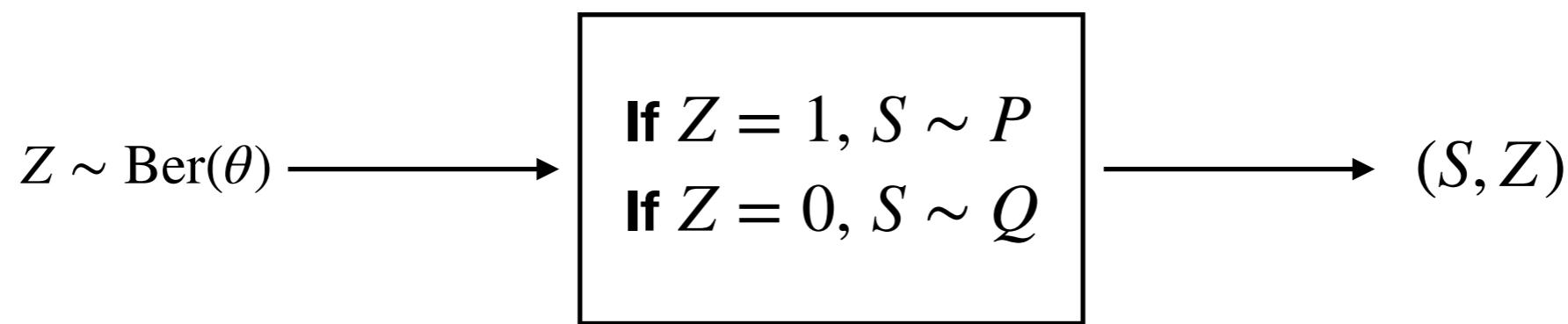
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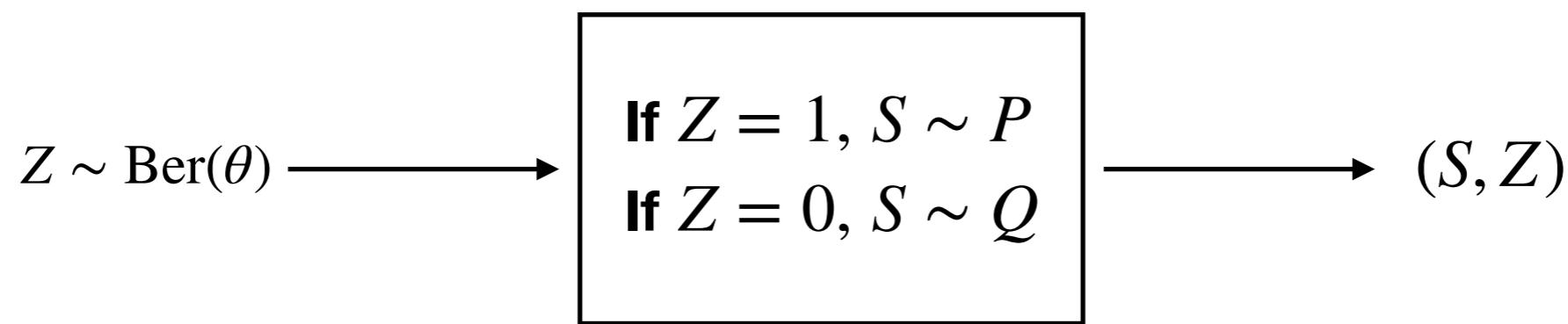
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A Different Perspective on Two-Sample Testing

Data Model



The two-sample testing problem can be recast as an **independence test** here.

$$H_0 : p(S | Z = 0) = p(S | Z = 1) \text{ or } S \perp\!\!\!\perp Z$$

$$H_1 : p(S | Z = 0) \neq p(S | Z = 1) \text{ or } S \not\perp\!\!\!\perp Z$$

Without Further Ado: The Bimodal Query Algorithm

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Bimodal Query Algorithm

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1. **Construct a training set:** Randomly select a set of features and reveal their labels

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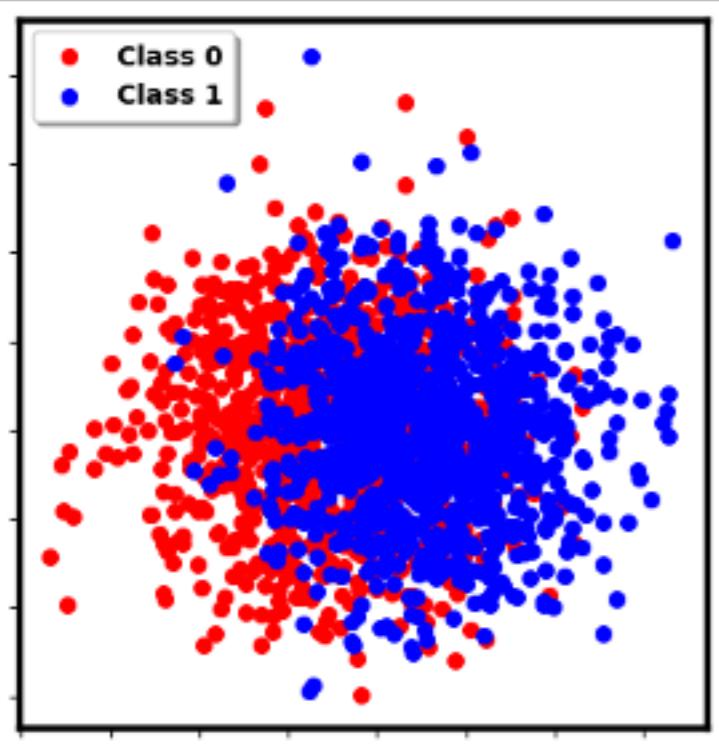
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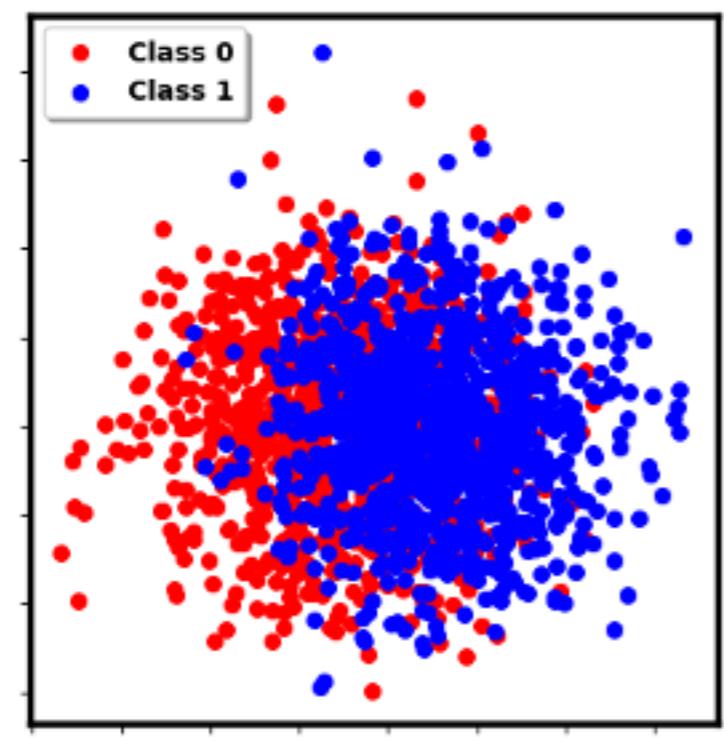
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4. **Two-sample testing:** Construct a two-sample test (e.g., FR test) on the resulting two samples

What does this do?

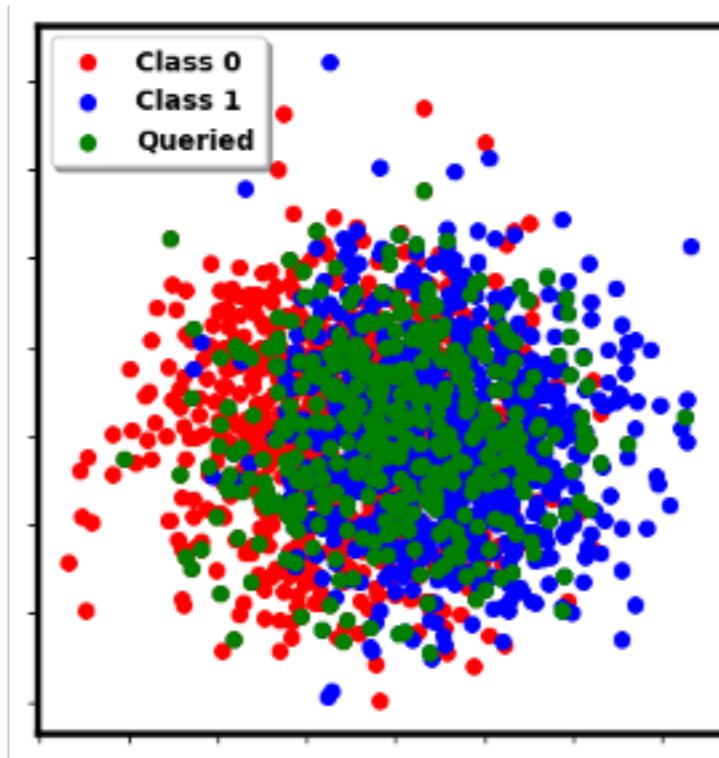


**Sample features (the labels
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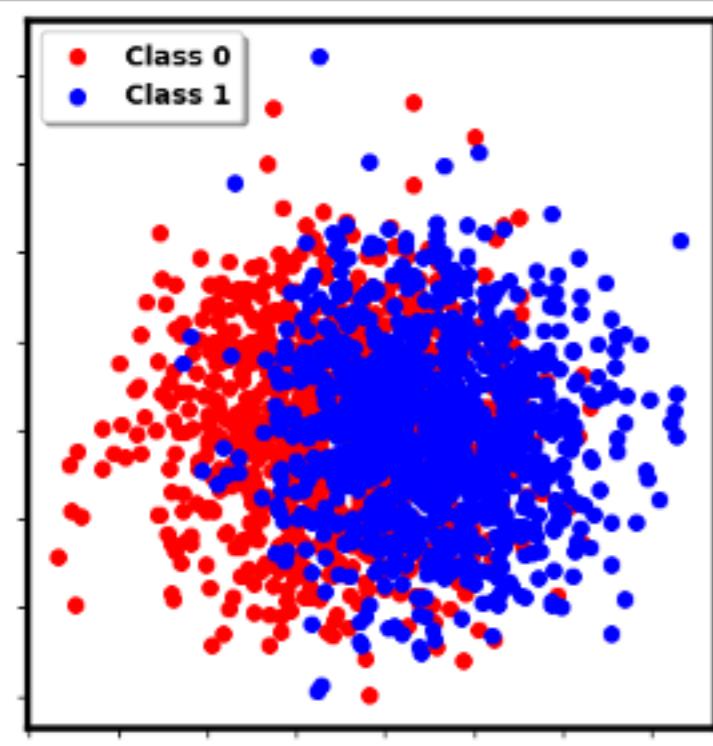


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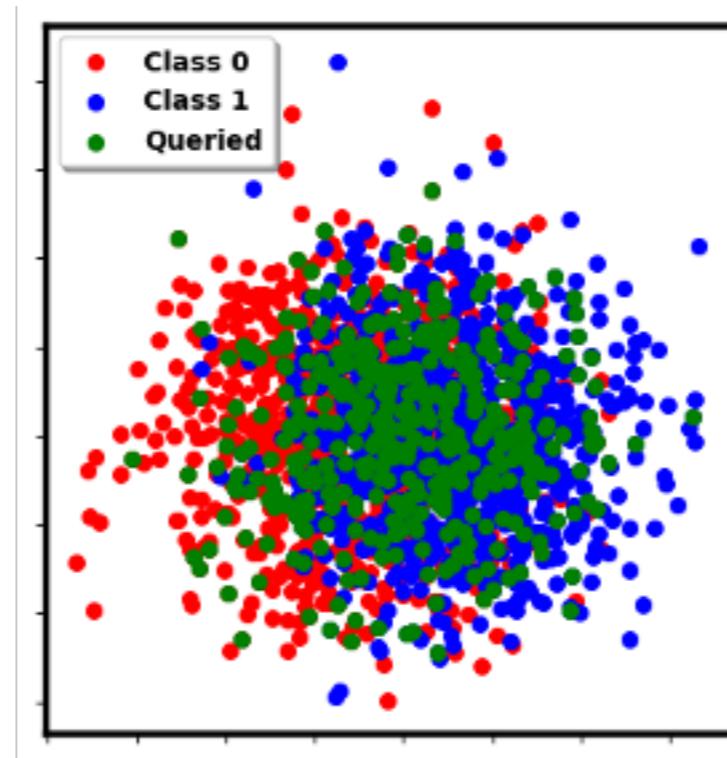


Random sampling

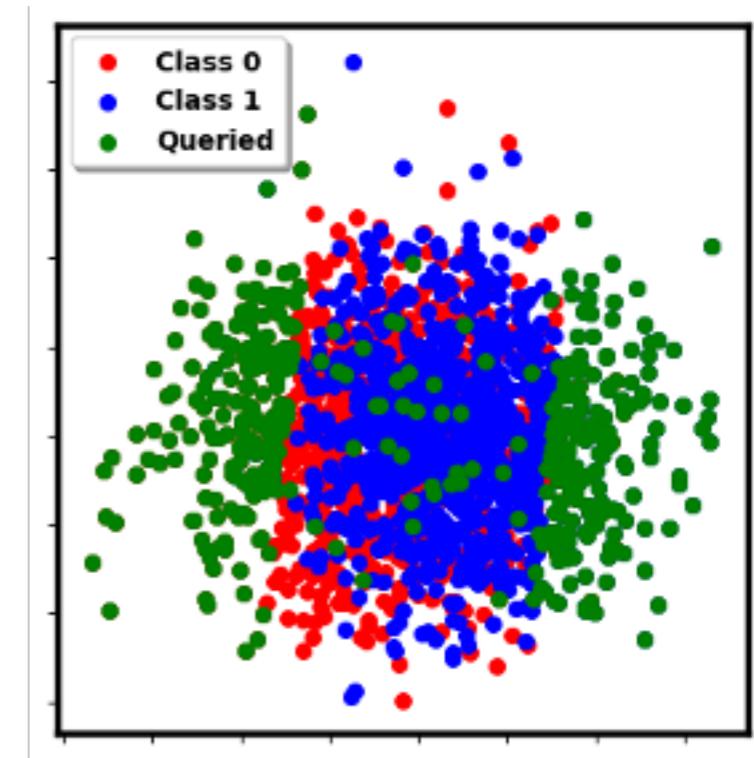
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Sample features (the labels
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Random sampling



Bimodal query

What does this *really* do?

Theorem (LKSРDB 24/LDRB 22). Assuming an **appropriate classifier (e.g., KNN) is used**, then under H_1 , the distribution of features s selected by the bimodal query **converges to $p^*(s)$** -- the distribution that makes the FR statistic **maximally powered**.

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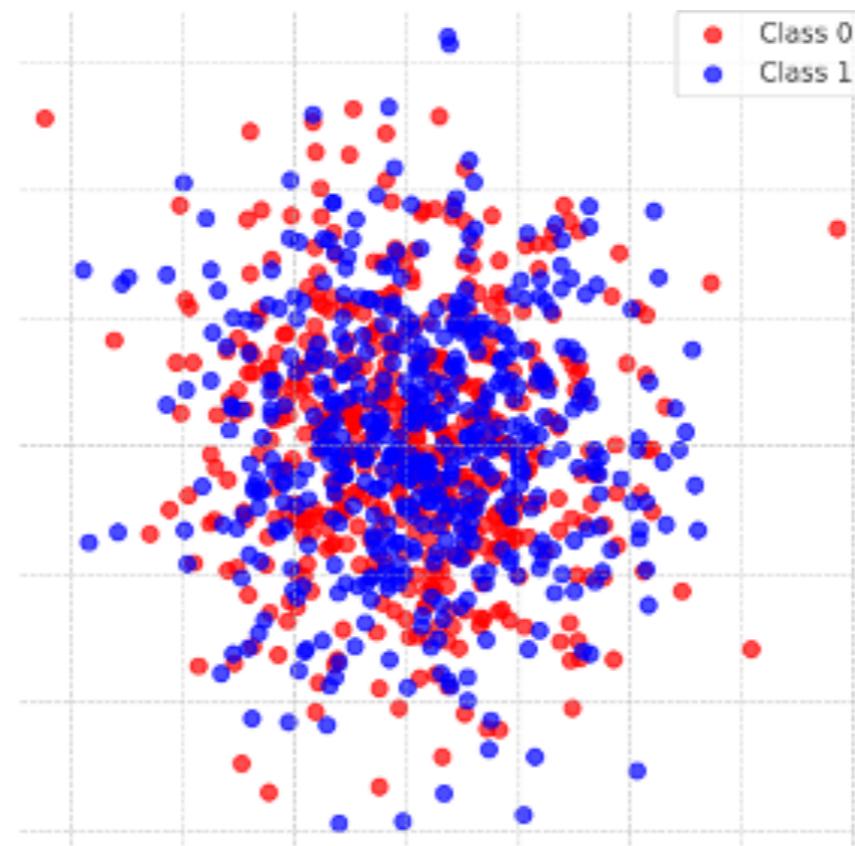
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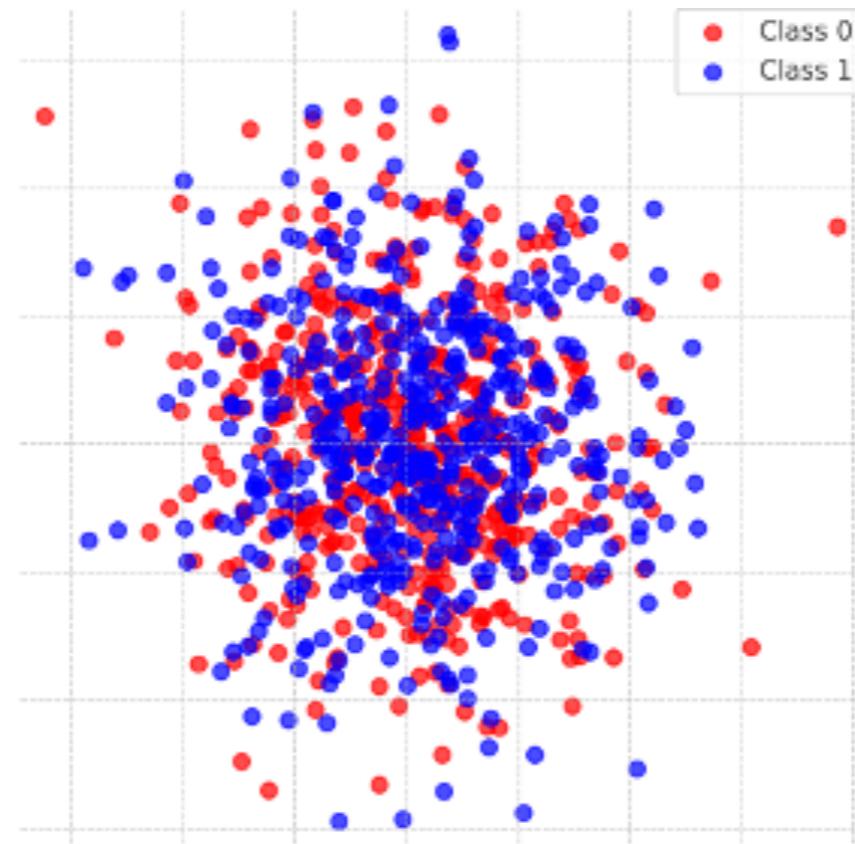
- We first show a structural result: the FR statistic (R/n) **converges to a function of** $\int p(Z = 0 | S)p(Z = 1 | S)dp(S)$.
- We then show that this function is **minimized (asymptotically)** by our Bimodal Query. LP in $p(s)$ —> optima at extremes, roughly.

Type I Error Control



Theorem 2 (LKS RDB 24). Under H_0 , $p(S \mid Z = 0)$ and $p(S \mid Z = 1)$ are identical. Consequently our procedure (built on, say FR) **controls the Type I error** at the specified level.

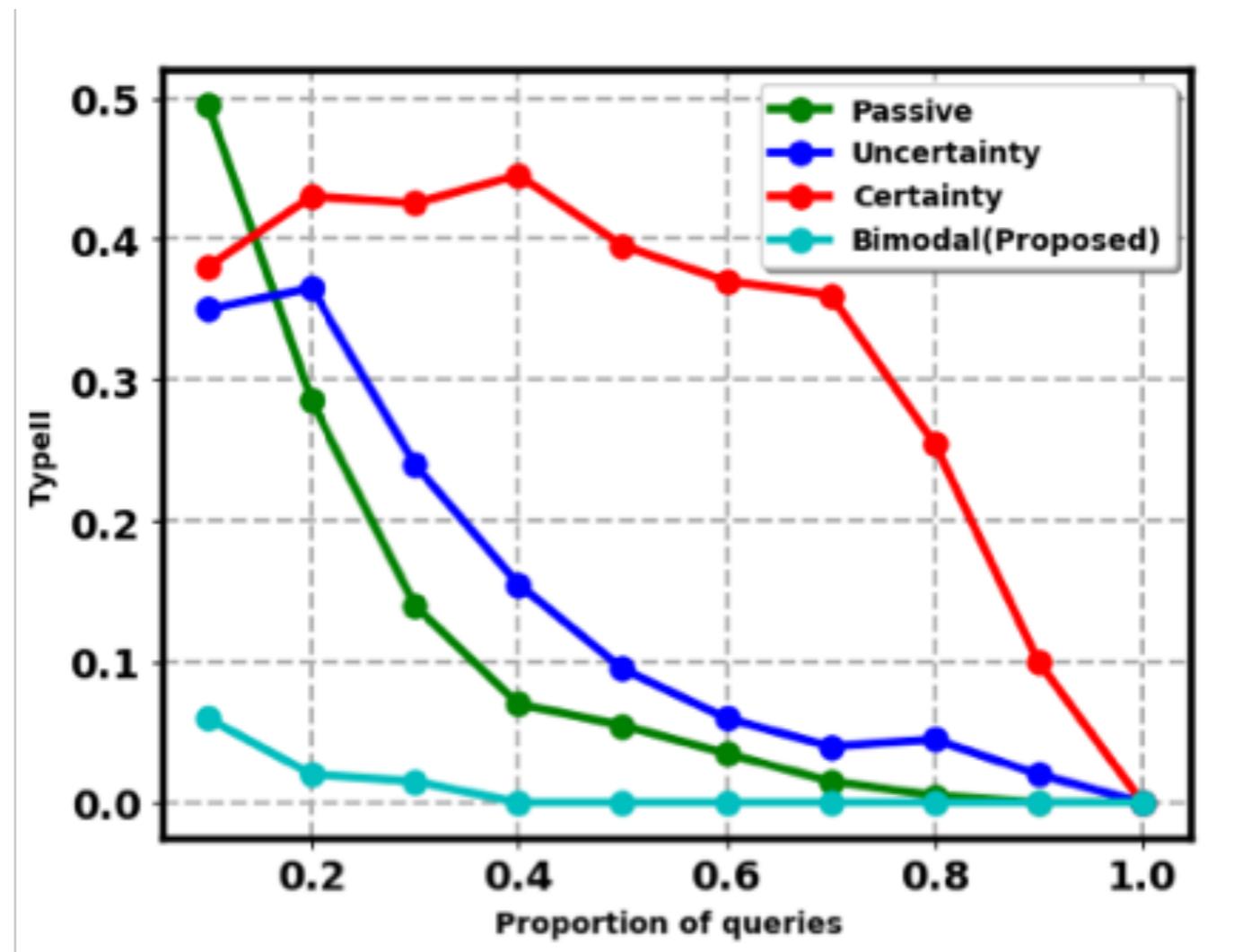
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Alzheimer's Disease Neuroimaging Initiative (ADNI)

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5 cognition measure scores + PET based Amyloid Scores

Beyond Fixed Budgets: Sequential and Design Extensions

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Sequential Two Sample Testing

- Label acq can be **done sequentially**
- Maintain a **running test-statistic**
- Produces **anytime-valid p-values** — Type-I control holds *even under adaptive stopping*
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Bridges **testing and design**: both are adaptive inference under verification constraints.



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Pouria Saidi
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Visar Berisha
ASU



CNS-2003111, and CCF-2048223



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