Design and Performance Analysis of Medium Access Control (MAC) Protocols for Multipacket Reception

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Multipacket Reception

Collision Channel

- Physical layer limitation
  - *More than one node access the channel simultaneously* $\Rightarrow$ *Collision*
  - (0,1,e) Feedback
- Protocols - IEEE 802.11, Aloha, Splitting tree

Capture and MPR

- Physical Layer Technologies
  - MUD - Multiuser detection
  - DS-CDMA - Code Division Multiple Access
  - MU-MIMO - Multiple Input Multiple Output

Problem statement

Design and analysis of MAC protocols for networks capable of Multipacket reception

ALOHA Analysis

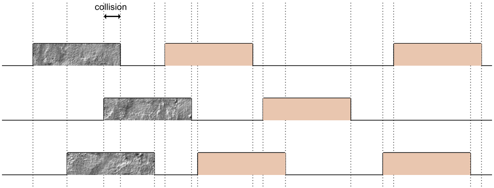


Figure: Packet collision, $K = 2$

- Channel Model
  - k-MPR, Generalized MPR
- Network Model
  - Infinite user model
    - Poisson packet arrivals
  - Fixed packet lengths
- Throughput (S)
  - ⋆ Time average of the number of packets successfully received
  - ⋆ Computation: $S = \Lambda \times \Pr($Success of a tagged packet$)$

ALOHA- Bounds on throughput

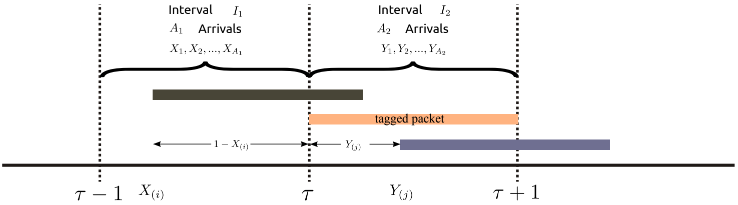


Figure: Tagged packet transmission

Lower Bound :
$$S_{\text{ALOHA}} \geq \Lambda \sum_{i=0}^{K-1} \frac{(2\Lambda)^i e^{-2\Lambda}}{i!} \quad (1)$$

Upper Bound :
$$S_{\text{ALOHA}} \leq \Lambda \left(\sum_{i=0}^{K-1} \frac{\Lambda^i e^{-\Lambda}}{i!} \right)^2 \quad (2)$$

$$S_{\text{slotted}} = \Lambda \left(\sum_{i=0}^{K-1} \frac{\Lambda^i e^{-\Lambda}}{i!} \right) \geq S_{\text{ALOHA}} \quad (3)$$

Vulnerable Interval

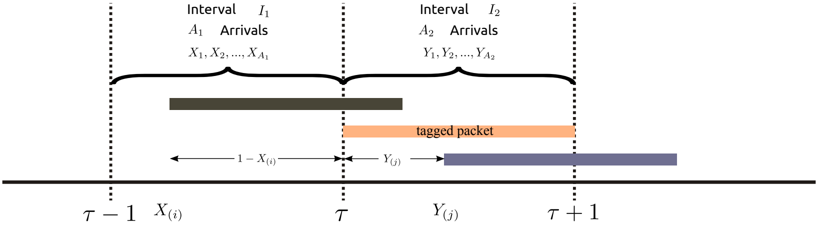


Figure: Tagged packet transmission

- Conditioned on $A_1 = a_1$, the arrival times are uniform in I_1.
- $X(Y)$: measured from the beginning of $I_1(I_2)$ is $\mathcal{U}(0,1)$.

$X_{(i)}$: i^{th} smallest from a set of a_1 uniform r.v. (i^{th} order statistic)

$Y_{(j)}$: j^{th} smallest from a set of a_2 uniform r.v. (j^{th} order statistic)

Conditions for non-overlapping

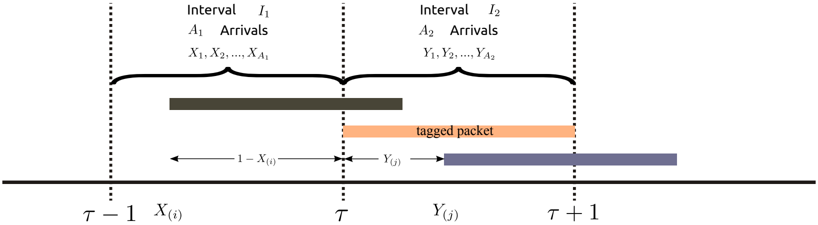


Figure: Tagged packet transmission

$\mathbb{S}_1 \equiv \{$ordered set of arrivals in $I_1\}$, $\mathbb{S}_2 \equiv \{$ordered set of arrivals in $I_2\}$

- Any n.o. pair can be written as $\langle l, m \rangle$, where $l \in \mathbb{S}_1$, and $m \in \mathbb{S}_2$.
- $X_{(i)}$ and $Y_{(j)}$ are n.o. $\Rightarrow 1 - X_{(i)} + Y_{(j)} > 1 \equiv X_{(i)} < Y_{(j)}$

ALOHA- Order statistics based Analysis

Definitions

- $\mathbb{D}$ is a maximal set of distinct non-overlapping pairs
- $D = |\mathbb{D}|$
- W : the maximum number of transmissions interfering with the tagged packet.

Lemma

$W = A - D$

Proof.

The effective interference from a non-overlapping pair of packets to the tagged node will be one (not two). Then, the number of transmissions, at any time, during interval I will be less than or equal to $A - D + 1$ [a], i.e. $W = A - D$. □

[a] D non-overlapping pairs + $(A - 2D)$ unpaired + 1 tagged

CMF of W

Lemma

$$F_W(w \mid a_1, a_2) = \Pr\left(\bigcap_{i=1}^{a-w} \{X_{(i)} < Y_{(w-a_1+i)}\}\right)$$

Proof.

The first d arrivals in I_1 should be non-overlapping with the last d arrivals in I_2 in that order □

Proof(Formal).

From previous Lemma, $A = a \Rightarrow W \leq w$ iff $D \geq a - w$.
$D \geq d \Rightarrow \langle i, a_2 - d + i \rangle \; \forall i = 1..d$, should be nonoverlapping.
$X_{(i)} \not< Y_{(a_2-d+i)} \Rightarrow$ any n.o. pair $\langle l, m \rangle$ should satisfy (i) $l < i$ or (ii) $m > a_2 - d + i$. □

Illustration

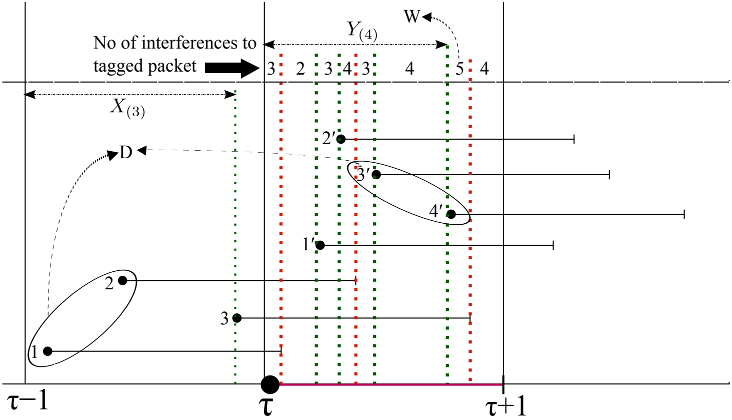


Figure: $\mathbb{S}_1 = \{1, 2, 3\}$, $\mathbb{S}_2 = \{1', 2', 3', 4'\}$, $A_1 = 3$, $A_2 = 4$, $A = 7$, $W = 5$. A maximal set of distinct non-overlapping pairs $\mathbb{D} = \{\langle 1, 3\rangle, \langle 2, 4\rangle\}$, therefore $D = |\mathbb{D}| = 2$. Note that $A - D = W = 5$. If $K \geq 6$ then $W \leq K - 1$ and tagged packet will be successful.

Probability of Success

Lemma

$$P_{suc}(a) = \frac{1}{2^a} \sum_{i=a-(K-1)}^{K-1} \binom{a}{i} F_W(K-1 \mid i, a-i) \quad \forall\; K-1 < a < 2K-1$$

Proof.

If $A_1 = i$ and $A_2 = a - i$, then the probability of success is $F_W(K-1 \mid i, a-i)$.

$$\Pr(A_1 = i, A_2 = a - i | A = a) = \frac{1}{2^a}\binom{a}{i} \tag{4}$$

$\because$ Each of the a arrivals in I is equally likely to fall in I_1 or I_2. □

Theorem

Throughput of pure ALOHA in a channel with MPR capability K is given by

$$S = \Lambda \left[\sum_{i=0}^{K-1} \frac{(2\Lambda)^i e^{-2\Lambda}}{i!} + \sum_{i=K}^{2K-2} \frac{\Lambda^i e^{-2\Lambda}}{i!} \sum_{j=i-(K-1)}^{K-1} \binom{i}{j} F_W(K-1 \mid j, i-j) \right]$$

Generalized MPR Channels

Theorem

Throughput of pure ALOHA under generalized MPR channel with reception matrix C is given by,

$$\Lambda \sum_{a=0}^{2K-2} \frac{\Lambda^a e^{-2\Lambda}}{a!} \sum_{i=0}^{a} \binom{a}{i} \sum_{j=\min(i,a-i)}^{K-1} \frac{\bar{R}_{j+1}}{j+1} f_W(j \mid i, a-i)$$

Proof.

$S = \sum_{i=0}^{2K-2} \Pr(A = a)\bar{p}(a)$, where $\bar{p}(a)$ is the conditional expectation of probability of success when $A = a$.

$\bar{p}(a) = \frac{1}{2^a} \sum_{i=0}^{a} \binom{a}{i} \sum_{j=\min(i,a-i)}^{K-1} f_W(j \mid i, a-i) \frac{\bar{R}_{j+1}}{j+1}$

□

Finite Nodes

- N nodes with arrival rates $\lambda_1, \lambda_2, \ldots \lambda_N$.
- Tagged packet does not suffer collision from another packet from the same node
- Aggregate traffic from other nodes approximated as Poisson.

$$S(\Lambda, K, N) = \frac{N}{N-1} S\left(\frac{N-1}{N}\Lambda, K\right) \tag{5}$$

$$S_i = \lambda_i \frac{S(\Lambda - \lambda_i, K)}{\Lambda - \lambda_i}$$

$$S(\lambda_1, \ldots, \lambda_N, N, K) = \sum_{i=1}^{N} S_i = \sum_{i=1}^{N} \lambda_i \frac{S(\Lambda - \lambda_i, K)}{\Lambda - \lambda_i} \tag{6}$$

Computation of $F_W(w \mid a_1, a_2)$

Direct Method

$$F_W(w \mid a_1, a_2) = \Pr\left(X_{(1)} \leq Y_{(a_2-d+1)}, X_{(2)} \leq Y_{(a_2-d+2)}, \ldots, X_{(d)} \leq Y_{(a_2)}\right) \tag{7}$$

$$= \mathrm{E}_{Y_{(\Phi)}}[F_{X_{(\Omega)}}(y_{(a_2-d+1)}, \ldots, y_{(a_2)})] \tag{8}$$

where, $\Omega = \{1, 2.., d\}, \Phi = \{a_2 - d + 1, \ldots, a_2\}$

$$F_{X_{(\Omega)}}(x_{(1)}, x_{(2)}, .., x_{(d)}) = \sum_{i_1=1}^{i_2} \sum_{i_2=2}^{i_3} \cdots \sum_{i_d=d}^{i_{d+1}} \left\{ a_1! \prod_{j=1}^{d+1} \left[\frac{(x_{(j)} - x_{(j-1)})^{i_j}}{(i_j - i_{j-1})!} \right] \right\} \tag{9}$$

$$f_{Y\,(\Phi)}(y_{(a_2-d+1)}, .., y_{(a_2)}) = \frac{a_2!}{(a_2 - d)!} (y_{(a_2-d+1)})^{a_2-d} \tag{10}$$

Using moments

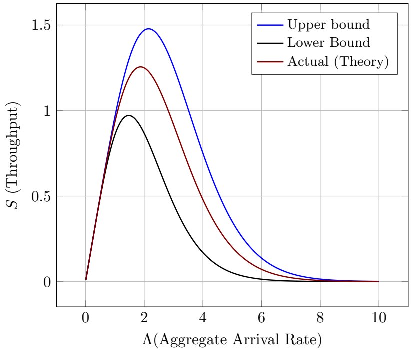
$$\mathrm{E}[\prod_{i=1}^{k} X_{(r_i)}^{a_i}] = \frac{n!}{(n + \sum_{i=1}^{k} a_i)!} \prod_{i=1}^{k} \frac{(r_i - 1 + \sum_{j=1}^{i} a_j)!}{(r_i - 1 + \sum_{j=1}^{i-1} a_j)!} \tag{11}$$

Results: Pure ALOHA throughput for MPR

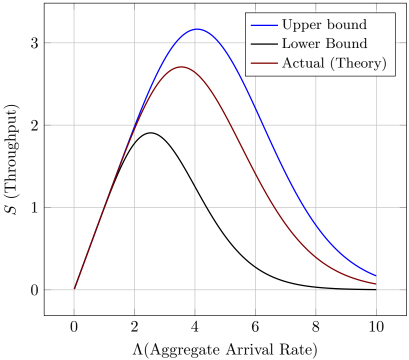
K	Throughput= $\Lambda e^{-2\Lambda}$ times the polynomial given below
2	$1+2\Lambda+\frac{1}{2}\Lambda^2$
3	$1+2\Lambda+2\Lambda^2+\frac{2}{3}\Lambda^3+\frac{1}{12}\Lambda^4$
4	$1+2\Lambda+2\Lambda^2+\frac{4}{3}\Lambda^3++\frac{11}{24}\Lambda^4+\frac{1}{12}\Lambda^5+\frac{1}{144}\Lambda^6$
5	$1+2\Lambda+2\Lambda^2+\frac{4}{3}\Lambda^3+\frac{2}{3}\Lambda^4+\frac{13}{60}\Lambda^5+\frac{2}{45}\Lambda^6+\frac{1}{180}\Lambda^7+\frac{\Lambda^8}{2880}$
6	$1+2\Lambda+2\Lambda^2+\frac{4\Lambda^3}{3}+\frac{2\Lambda^4}{3}+\frac{4\Lambda^5}{15}+\frac{19\Lambda^6}{240}+\frac{\Lambda^7}{60}+\frac{7\Lambda^8}{2880}+\frac{\Lambda^9}{4320}+\frac{\Lambda^{10}}{86400}$
7	$1+2\Lambda+2\Lambda^2+\frac{4}{3}\Lambda^3+\frac{2}{3}\Lambda^4+\frac{4\Lambda^5}{15}+\frac{4\Lambda^6}{45}+\frac{\Lambda^7}{42}+\frac{11\Lambda^8}{2240}+\frac{23\Lambda^9}{30240}+\frac{13\Lambda^{10}}{151200}+\frac{\Lambda^{11}}{151200}+\frac{\Lambda^{12}}{10!}$

Table: Throughput of pure ALOHA for K=2 to 7

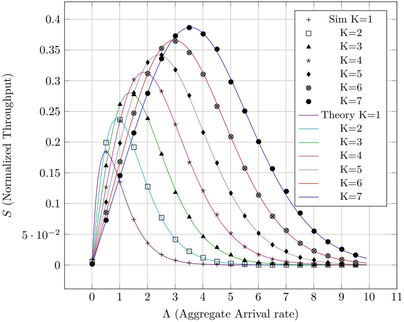
ALOHA: Simulation Results



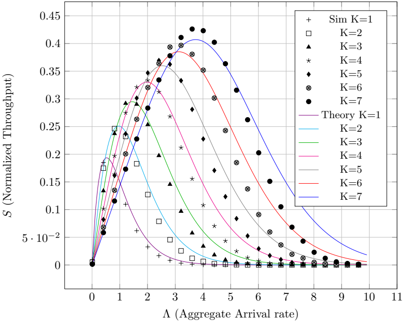
(a) Throughput of ALOHA against channel traffic ($K = 4$)



(b) Throughput of ALOHA against channel traffic ($K = 7$)



(c) The throughput of ALOHA $N = \infty$



(d) The throughput of ALOHA, $N = 20$

Arun IB, T.G.Venkatesh, "*Order statistics based analysis of Pure ALOHA in channels with Multipacket Reception*", IEEE Communication Letters,Vol.17, no.10,October 2013

Non-persistent CSMA with MPR

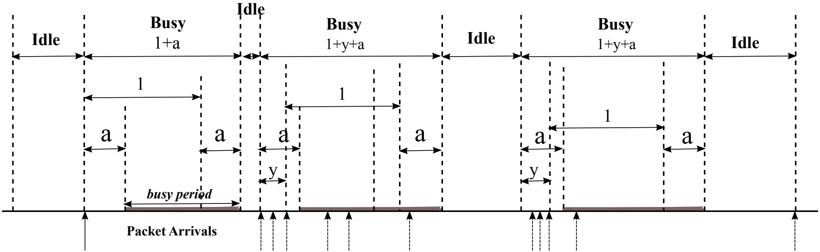


Figure: Illustrating the channel and time of NP-CSMA. Arrivals to a busy period are scheduled for transmission after a *random* time

Non-persistent CSMA with MPR

$$S = \frac{\sum_{i=0}^{K-1} (i+1)(\Lambda a)^i e^{-\Lambda a}}{\frac{1}{\Lambda} + 1 + 2a - \frac{1}{\Lambda}(1 - e^{-a\Lambda})} \quad (12)$$

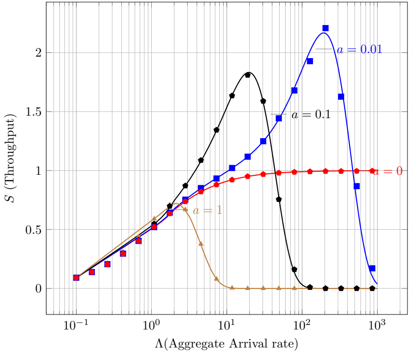


Figure: Throughput of non-persistent CSMA with MPR limit $K = 4$: Theory(lines) vs Simulation(symbols)

Adaptive MPR CSMA Protocol

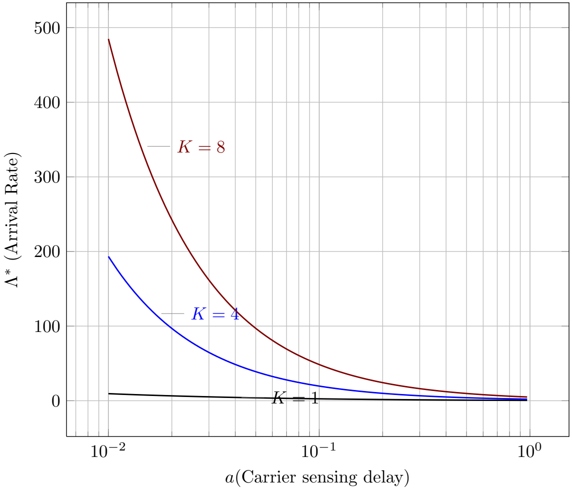


Figure: Effect of carrier sensing delay on arrival rate at which throughput is maximum

1-persistent CSMA with MPR

The three kinds of transmission periods:

- An idle transmission period (TP) called Type 0.
- A TP which starts with the transmission of a single packet (Type 1) and which follows the type-0 transmission period.
- A type 2 TP follows an arrival into a busy channel. A type-2 transmission may begin with more than one packet transmission.

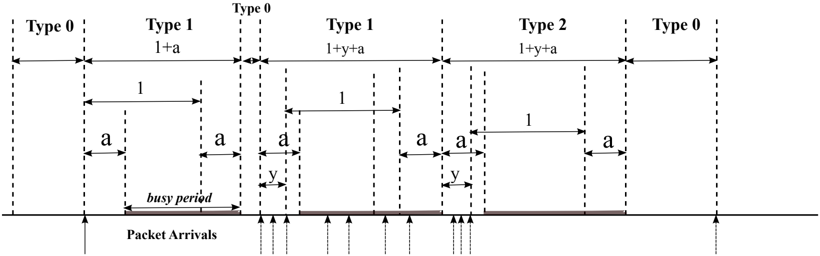


Figure: Illustrating the channel and time: 1P-CSMA. Arrivals to a busy period are scheduled for transmission at the end of the current TP

1-persistent CSMA with MPR

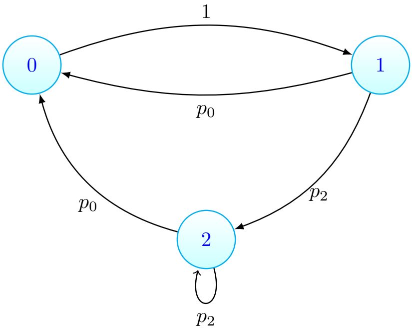


Figure: Markov chain for the transmission periods (TP) of 1-persistent CSMA Protocol with MPR.

$$S = \frac{\pi_i \hat{S}_i}{\sum_{i=0}^{2} \pi_i E[T_i]} \quad (13)$$

1-persistent CSMA with MPR

$$\pi_0 = \pi_1 = \frac{(1+a\Lambda)e^{-\Lambda(1+a)}}{1+(1+a\Lambda)e^{-\Lambda(1+a)}} \quad (14)$$

$$\pi_2 = \frac{1-(1+a\Lambda)e^{-\Lambda(1+a)}}{1+(1+a\Lambda)e^{-\Lambda(1+a)}} \quad (15)$$

$$\hat{S}_1 = \sum_{i=0}^{K-1} (i+1)(a\Lambda)^i \frac{e^{-a\Lambda}}{i!} \quad (16)$$

$$E[\hat{S}_2] = e^{-a\Lambda} \sum_{i=1}^{K} \sum_{j=0}^{K-i} (i+j) \frac{a^j \Lambda^{i+j} \frac{e^{-\Lambda}}{i!} \frac{e^{-a\Lambda}}{j!}}{1-e^{-\Lambda}} + \sum_{i=1}^{K} \sum_{j=0}^{K-i} \frac{(i+j)a^j \Lambda^{i+j} e^{-(1+2a)\Lambda}}{i!j!} \int_{y=0}^{a} \frac{(1+y)^i}{1-e^{-(1+y)\Lambda}} dy \quad (17)$$

1-persistent CSMA with MPR

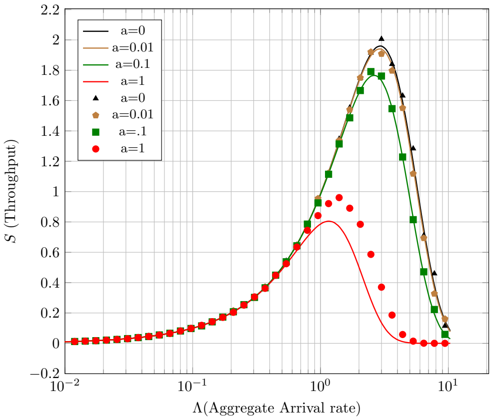


Figure: Throughput of 1P-CSMA with MPR: Analysis(lines) vs Simulation (symbols) for $K = 4$

Adaptive Backoff algorithm for IEEE 802.11 under MPR

Parameters

- K: MPR capability
- i: No of ongoing transmissions
- d(i): the value of counter decrements in a slot for a given i
- $K_t (< K)$: Threshold

Adaptive MAC protocol

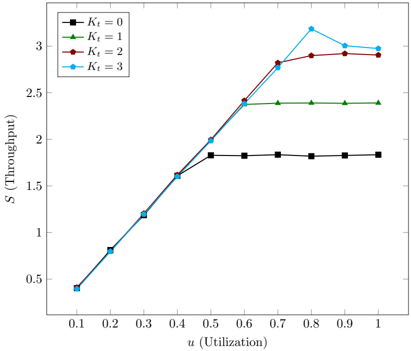
$$d(i) = \begin{cases} K - i & i \leq K_t \\ 0 & otherwise \end{cases}$$

Proposed protocol

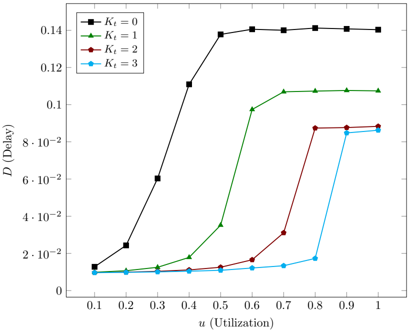
- Decrement the counter based on channel utilization $(K - i)$
- DIFS - wait till number of ongoing transmissions go below K_t

When the channel utilization is low, the counter gets decremented faster and nodes attempt transmissions sooner $\Rightarrow$ improved delay and throughput

Simulation Results



(a) The throughput of proposed protocol against normalized channel utlization



(b) The average MAC delay of proposed protocol against normalized channel utlization

Figure: Proposed protocol performance for different thresholds (*Params*: $K = 4$, N = 30, $CW_{min} = 128$, $m = 5$, $rlimit = 4$)

Performance Comparison

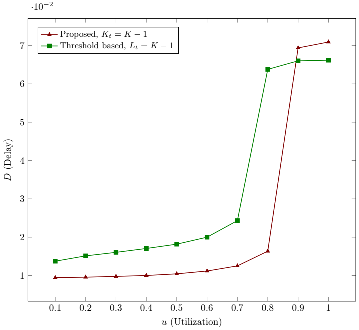


Figure: The MAC delay of proposed protocol and threshold based protocol against utilization(*Params*: $K = 7$, N=30, $CW_{min} = 128$, $m = 5$)

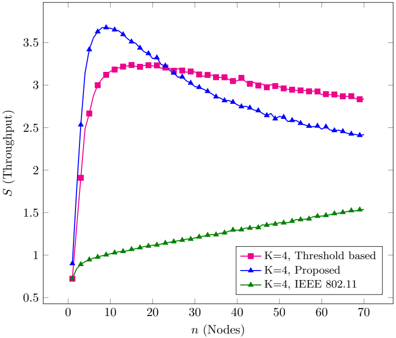


Figure: The normalized saturation throughput of different protocols against number of nodes.(*Params*: $K = 4$, $CW_{min} = 128$)

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Analysis of generalized counter decrements

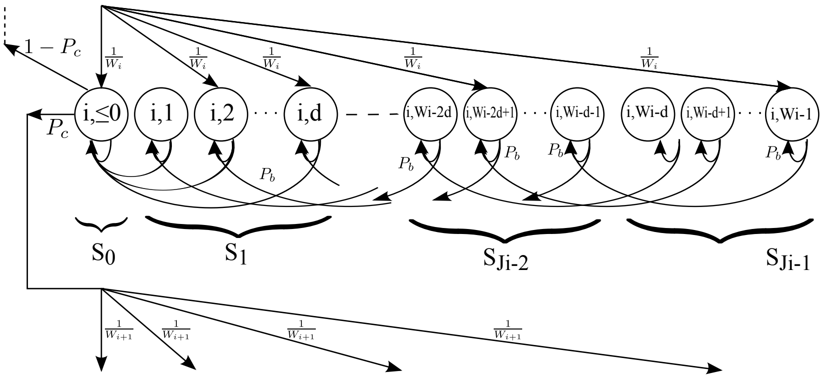


Figure: One stage of the Markov Chain for the backoff process of DCF for the case of backoff decrements by a value $d \geq 1$

$$S_k = \{(k-1)d + 1, \ldots, kd\} \quad \forall k \in [0, J_i - 1]$$

$$J_i - 1 = \left\lfloor \frac{W_i - 1}{d} \right\rfloor$$

Markov Chain for backoff process of MRP DCF

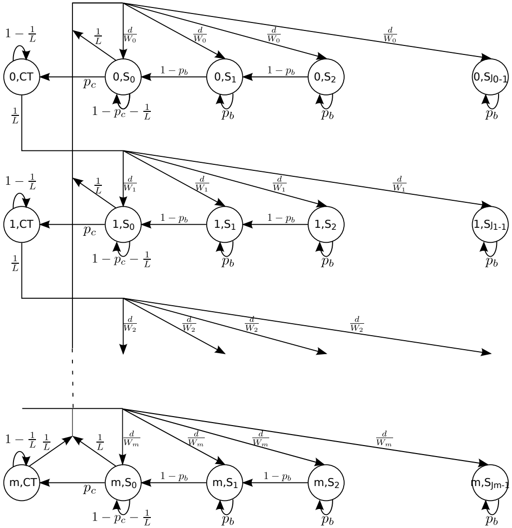


Figure: Markov Chain for the backoff process MPR DCF for uniform counter decrementing by a value $d \geq 1$

$$r = \frac{Lpc}{1 + Lp_c} \tag{18}$$

$$b(i, CT) = Lp_c r^i b(0, S_0) \quad \forall\, i \in [0, m] \tag{19}$$

$$b(i, S_k) = \frac{J_i - k}{J_i} \frac{1}{1 - p_b} r^{(i-1)} b(0, S_0) \\ \forall\, i \in [1, m],\ k \in [1, J_i - 1] \tag{20}$$

$$b(0, S_0) = \frac{1}{(1 + Lp_c)(\frac{1-r^{m+1}}{1-r}) + \frac{p_c}{1-p_b}(\frac{1-(2r)^m}{1-2r} J_0 - \frac{1}{2}\frac{1-r^m}{1-r})} \tag{21}$$

$$\tau_d = \frac{(\frac{1}{L} + p_c)\frac{1-r^{m+1}}{1-r}}{(1 + Lp_c)(\frac{1-r^{m+1}}{1-r}) + \frac{p_c}{1-p_b}(\frac{1-(2r)^m}{1-2r}\left\lceil \frac{W_0}{d} \right\rceil - \frac{1}{2}\frac{1-r^m}{1-r})} \tag{22}$$

Channel state Markov Chain

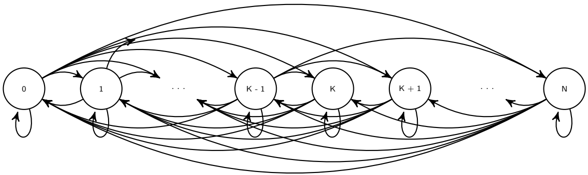


Figure: Channel state DTMC; States → (# of ongoing transmissions)

- Each node starts transmission with probability τ
- $q(i,j)$ is the probability that j additional nodes starts transmission in a slot in which i transmissions are going on
- $r(i,j)$ is the probability that j out of i ongoing transmissions finish
- At every slot, an ongoing transmission encounters a collision with probability p_c
- Duration of a single packet transmission is geometrically distributed with mean L slots

Transitions

$$q(i,j) \equiv \binom{N-i}{j} \tau^j (1-\tau)^{N-i-j}$$

$$r(i,j) \equiv \binom{i}{j} \left(\frac{1}{L}\right)^j \left(1-\frac{1}{L}\right)^{i-j}$$

State Transitions

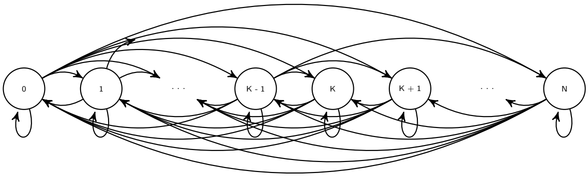


Figure: Channel state DTMC; States → (# of ongoing transmissions)

Forward transitions

$$p_{i,i+m} = \sum_{j=0}^{\min(i,N-i)} q(i, m+j) r(i, j) \quad \forall i \leq K_t \tag{23}$$

$$p_{i,i+m} = 0 \quad \forall i > K_t, m > 0 \tag{24}$$

$$p_{i,i-m} = \sum_{j=0}^{\min(i,N-i)} r(i, m+j) q(i, j) \quad \forall i \leq K_t \tag{25}$$

$$p_{i,i-m} = r(i, m) \quad \forall i > K_t \tag{26}$$

Stationary Probabilities

$\pi = \pi P$

$\pi =$ left eig(P)

$\pi_i = |M_{i+1}|$ where M_j is the jth principal minor of P.

Throughput

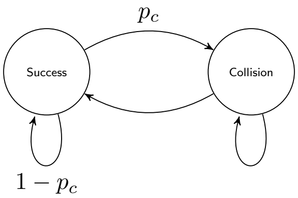


Figure: Two state DTMC of channel

Collision and Busy probabilities

$$p_c = \frac{\sum_{i=0}^{K} \sum_{j=K+1}^{N} \pi_i p_{i,j}}{\sum_{i=0}^{K} \pi_i} \qquad (27)$$

$$p_b = \sum_{i=K_t}^{N} \pi_i \qquad (28)$$

Solve

$$\tau = (\frac{1}{L} + p_c)\frac{1 - r^{m+1}}{1 - r} b(0,0) \qquad (29)$$

$$\text{where } r = \frac{Lp_c}{1 + Lp_c}$$

Throughput

Conditional collision probability,

$$\Pr(\text{Collision/tx}) = 1 - (1 - p_c)^L = p \qquad (30)$$

Normalized throughput S,

$$S = \frac{N\tau(1-p)}{\sigma} \qquad (31)$$

Channel State Transitions- Proposed protocol

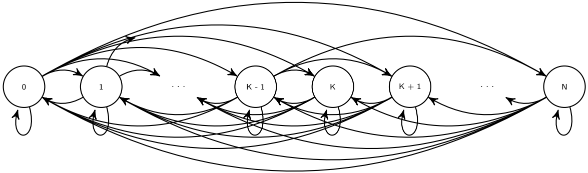


Figure: Channel state DTMC; States → (# of ongoing transmissions)

τ is function of i

$$q(i,j) \equiv \binom{N-i}{j}\tau(i)^j(1-\tau(i))^{N-i-j} \quad (32)$$

$$r(i,j) \equiv \binom{i}{j}(\frac{1}{L})^j(1-\frac{1}{L})^{i-j} \quad (33)$$

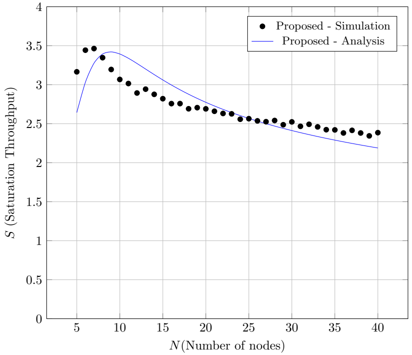
$\tau(i)$

If we define, $\tau(K-1) = \tau_0$ then

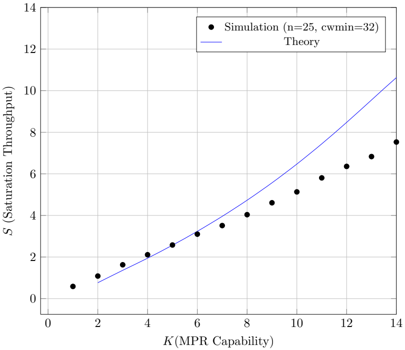
$$\tau(i) = (K-i)\tau_0$$

$$\tau = \sum_{i=0}^{K-1} \pi_i \tau(i) \quad (34)$$

Proposed Protocol: Analysis vs Simulation



(a) Throughput vs number of nodes N(Params: $K = 5, CW_{min} = 32$)



(b) Throughput vs MPR capability K (Params: $N = 30, CW_{min} = 32$)

Figure: Proposed protocol performance: theory vs simulation

- Enhancement to the IEEE 802.11ac EDCA protocol
- For MPR, in addition to CWmin, CWmax, and AIFSN, two more parameters namely (i) threshold and (ii) Counter decrementation value offers service differentiation

Table: Access categories and service differentiation

Access Category	Parameters
AC_0 Real time Voice	Kt $=$ K-1, Adaptive count down K - L
AC_1 Video playback	Kt $= \lceil K/2 \rceil$, Adaptive count down K - L
AC_2 Best effort	Kt $= \lceil K/4 \rceil$, Non-adaptive count down, Always decrement by 1
AC_3 File transfer	Kt $=$ 1 , Non-adaptive count down, Always decrement by 1

where Kt is threshold, K is MPR limit and L is estimated no.of transmission

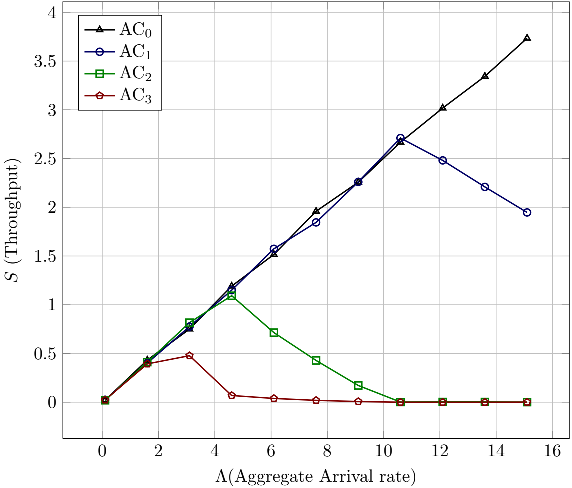


Figure: The throughput against offered traffic for different access categories (*Params*: Number of stations $N = 40$, $K = 8$, $m = 7$,$CW_{min} = 256$)

Adaptive Backoff Algorithm for IEEE 802.11ac - MPR

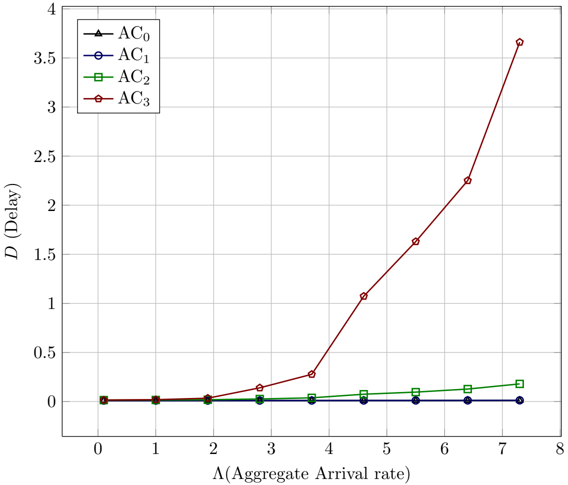


Figure: The MAC delay against offered traffic for different access categories (*Params*: Number of stations $N = 40$, $K = 8$)