Low Complexity Optimal Policies for Networked Control Systems

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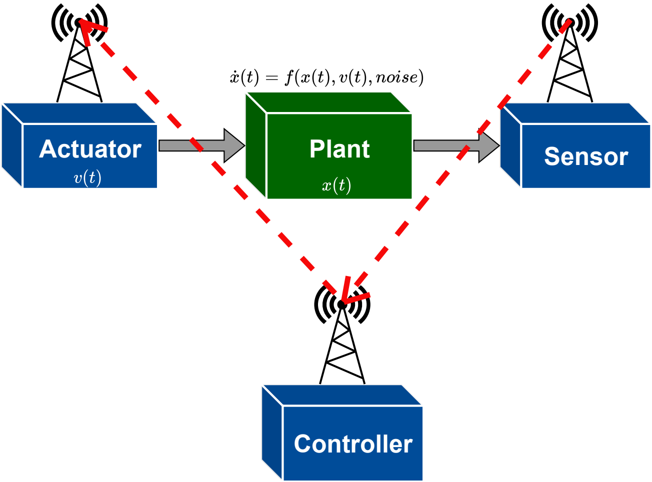
- Focus on Wireless Networked Control Systems (WNCS)



Industrial automation

Source:https://www.digi.com/

Traffic control

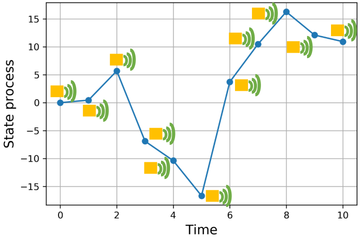
Source:https://www.digi.com/

Smart grid

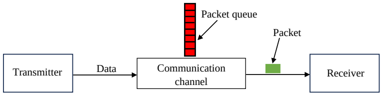
Source:https://www.energysage.com/

Motivation

- Transmission is expensive and consumes energy
- Continual transmissions are not efficient



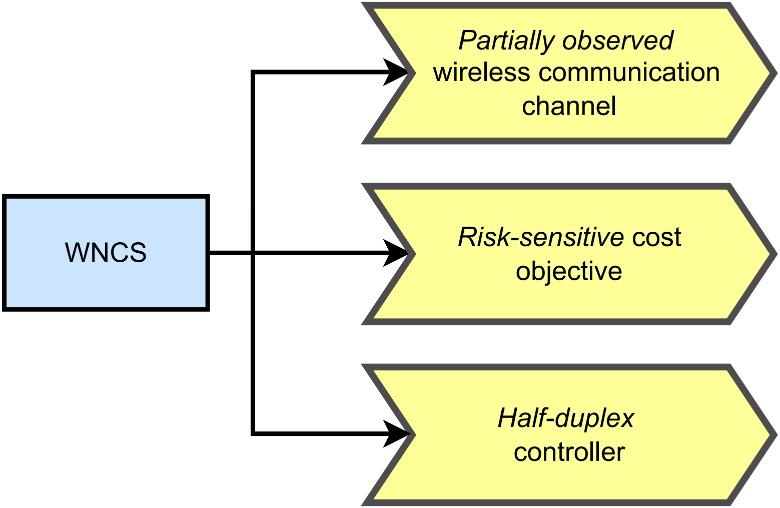
Continual transmissions



Channel congestion

WNCS Challenges

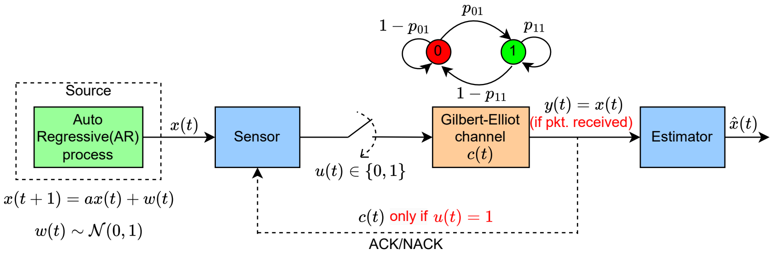
Consider problems



Optimal scheduling policies for remote estimation of autoregressive Markov processes over time-correlated fading channel

(2023 62nd IEEE Conference on Decision and Control (CDC) (pp. 6455-6462))

Partially observed channel



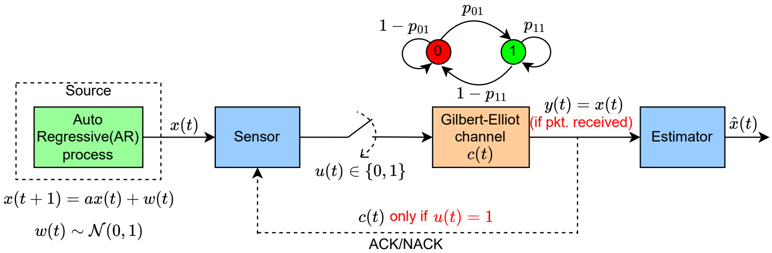
Sensor Channel state partially observed by the sensor

$$u(t) = \phi_t(\mathcal{F}(t)) \in \{0,1\} \text{ where,}$$

$\mathcal{F}(t)$ is the information available till time t

Channel model $c(t)$ Markovian; $c(t) = 1$: Good, $c(t) = 0$: Bad.

Partially observed channel



Estimator $\hat{x}(t) = \begin{cases} x(t) & \text{if pkt. received,} \\ a\hat{x}(t-1) & \text{otherwise} \end{cases}$

Estimation error $x(t) - \hat{x}(t)$

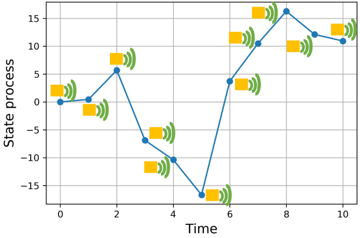
Transmission strategy $\phi = \{\phi_t\}_{t=0}^{\infty}$

Problem formulation

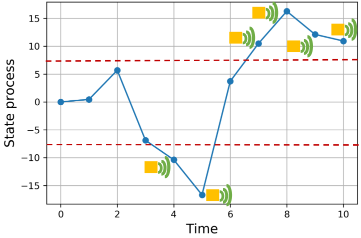
Goal

Consider problem:

- Optimal dynamic scheduling of sensor packet transmissions
- Trade-off between communication cost and estimation error



Continual transmissions



Dynamic transmissions

Optimization Problem

Problem

$$\min_{\phi} \ \mathbb{E}_{\phi} \left(\sum_{t=0}^{\infty} \beta^t \left((x(t) - \hat{x}(t))^2 + \lambda u(t) \right) \right),$$

discount factor $\beta \in (0,1)$, $\lambda u(t)$ is the communication cost
$\lambda > 0, u(t) \in \{0,1\}$

Literature Overview

Markovian communication channel

- [Ren et al., 2017]
  - Channel state instantaneously known to sensor
  - Optimal transmission strategy threshold-type w.r.t. error
- [Chakravorty and Mahajan, 2017; Chakravorty and Mahajan, 2019]
  - Channel state perfectly known to sensor with delay of one unit
  - Optimal transmission strategy threshold-type w.r.t. error

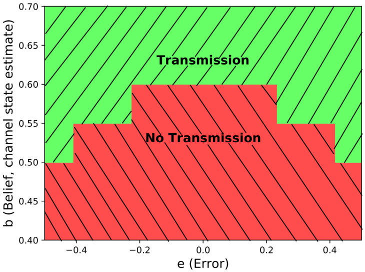
Key differences in our model

- Channel state partially observed by sensor
- Channel state known to sensor only via ACK sent by estimator when there is transmission attempt

Contributions

Our contributions

- Formulate optimization problem as a Partially Observable MDP(POMDP)
- Identify a dynamic programming decomposition
- Introduce “folded POMDP” to ease analysis
- Existence of an optimal transmission strategy exhibiting threshold structure w.r.t. belief state (channel state estimate)



POMDP Formulation

State **Belief state** $b(t) := \mathbb{E}(c(t)|\mathcal{F}(t))$,

where $\mathcal{F}(t)$ is the information available till time t

Error $e(t) = x(t) - a\hat{x}(t-1)$

Control $u(t) = \phi_t(e(t), b(t)) \in \{0, 1\}$

Instantaneous cost

$$d(e, b, u) := \begin{cases} a^2e^2 + \lambda u & \text{if } u = 0, \\ (1-b)(a^2e^2+1) + \lambda & \text{if } u = 1 \end{cases}$$

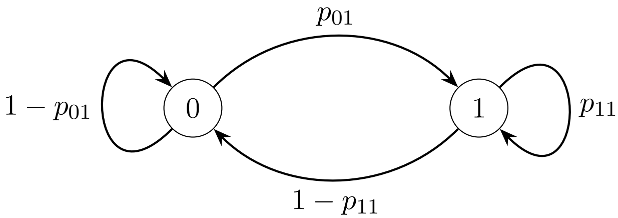
Transmission strategy $\phi = \{\phi_t\}_{t=0}^{\infty}$

POMDP

$$\min_{\phi} \ \mathbb{E}_{\phi}\left(\sum_{t=0}^{\infty} \beta^t (d(e(t), b(t), u(t)))\right)$$

Technical Assumptions

AR process: $x(t+1) = ax(t) + w(t)$



Gilbert-Elliot channel

(A1) Stability: $a^2(1 - p_{01}) < 1$

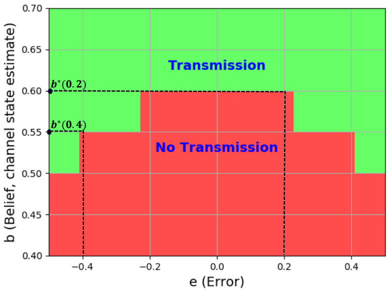
(A2) Positively correlated channel: $p_{11} \geq p_{01}$

Theorem 1: Structure of optimal transmission strategy

There exists a threshold-type optimal strategy,

$$u(t) = \phi^\star(e(t), b(t)) = \begin{cases} 1 & \text{if } b(t) \geq b^\star(|e(t)|), \\ 0 & \text{if } b(t) < b^\star(|e(t)|) \end{cases}$$

where $b^\star(|e(t)|)$ is threshold



Optimal strategy

Proof Outline

Step 1 Value iteration to solve the POMDP

Key challenge

- Instantaneous cost is unbounded
- State-space consists of error taking negative values

Step 2 Equivalent simpler folded POMDP

Step 3 Optimal transmission strategy for folded POMDP

Step 4 Unfolding

Step 1: Value Iteration

$$V^{(\beta)}(e,b) := \min_{\phi} \mathbb{E}_{\phi} \left(\sum_{t=0}^{\infty} \beta^t d(e(t), b(t), u(t)) \right)$$

Value iterates

- $V_0^{(\beta)}(e,b) = 0$
- $V_{n+1}^{(\beta)} = \min_{u \in \{0,1\}} Q_{n+1}^{(\beta)}(e,b;u)$ where,

  $Q_{n+1}^{(\beta)}(e,b;u) = d(e,b,u) + \beta \mathbb{E}_{e_+,b_+ \sim p(\cdot,\cdot|e,b;u)} [V_n^{(\beta)}(e_+,b_+)]$

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$$V^{(\beta)}(e,b) := \min_{\phi} \mathbb{E}_{\phi} \left(\sum_{t=0}^{\infty} \beta^t d(e(t), b(t), u(t)) \right)$$

Value iterates

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- $\lim_{n \to \infty} V_n^{(\beta)}(e,b) = V^{(\beta)}(e,b)$

Step 1: Value Iteration

$$V^{(\beta)}(e,b) := \min_\phi \mathbb{E}_\phi \left(\sum_{t=0}^{\infty} \beta^t d(e(t), b(t), u(t))\right)$$

Shows existence of optimal policy

- $V^{(\beta)}$ satisfies:

$$V^{(\beta)}(e,b) = \min_{u \in \{0,1\}} Q^{(\beta)}(e,b;u),$$

where,

$$Q^{(\beta)}(e,b;u) = d(e,b,u) + \beta \mathbb{E}_{e_+, b_+ \sim p(\cdot,\cdot|e,b;u)} \left[V^{(\beta)}(e_+, b_+) \right]$$

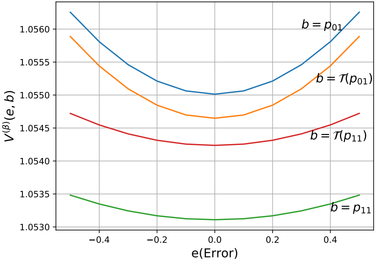
- $\phi^\star(e,b) \in \operatorname{argmin}_{u \in \{0,1\}} Q^{(\beta)}(e,b;u)$ is optimal

Step 2: Simpler Folded POMDP

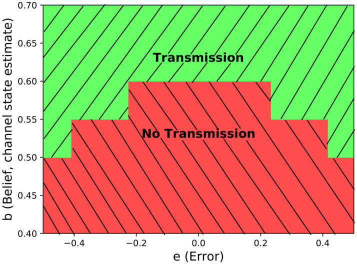
Even POMDP

For every $b \in [0,1], u \in \{0,1\}$, $V^{(\beta)}(e,b)$ and any optimal policy $\phi^\star(e,b)$ are even in e, i.e.,

- $V^{(\beta)}(e,b) = V^{(\beta)}(-e,b)$
- $\phi^\star(e,b) = \phi^\star(-e,b)$



$V^{(\beta)}$ even in e



Optimal policy

Step 2: Simpler Folded POMDP

State-space original POMDP $\mathbb{R} \times [0,1]$

folded POMDP $\mathbb{R}_+ \times [0,1]$

Folded POMDP

$e, e_+ \in \mathbb{R}_+$

$$p_{fold}(e_+, b_+ \mid e, b; u) = p(e_+, b_+ \mid e, b; u) + p(-e_+, b_+ \mid e, b; u)$$

Step 2: Simpler Folded POMDP

Equivalence result

$Q^{(\beta)}_{fold}, V^{(\beta)}_{fold}, \phi^{\star}_{fold}$ match with $Q^{(\beta)}, V^{(\beta)}, \phi^{\star}$ of the original POMDP i.e.,

- $Q^{(\beta)}(e, b; u) = Q^{(\beta)}_{fold}(|e|, b; u)$,
- $V^{(\beta)}(e, b) = V^{(\beta)}_{fold}(|e|, b)$,
- $\phi^{\star}(e, b) = \phi^{\star}_{fold}(|e|, b)$

- Suffices to consider folded POMDP (that is simpler is analyze)

Main Theorem (folded POMDP)

$V_{fold}^{(\beta)}$ satisfies:

(A) For each b, $V_{fold}^{(\beta)}(e, b)$ is non-decreasing in $e \in \mathbb{R}_+$

Step 3: Optimal Policy For Folded POMDP

Main Theorem (folded POMDP)

$V_{fold}^{(\beta)}$ satisfies:

(A) For each b, $V_{fold}^{(\beta)}(e, b)$ is non-decreasing in $e \in \mathbb{R}_+$

(B) For each $e \in \mathbb{R}_+$, $V_{fold}^{(\beta)}(e, b)$ is non-increasing in b

Step 3: Optimal Policy For Folded POMDP

Main Theorem (folded POMDP)

$V_{fold}^{(\beta)}$ satisfies:

(A) For each b, $V_{fold}^{(\beta)}(e,b)$ is non-decreasing in $e \in \mathbb{R}_+$

(B) For each $e \in \mathbb{R}_+$, $V_{fold}^{(\beta)}(e,b)$ is non-increasing in b

(C) For each $e \in \mathbb{R}_+$, there exits a threshold $b^\star(e)$ s.t.,

$$u = \begin{cases} 1 & \text{if } b \geq b^\star(e), \\ 0 & \text{if } b < b^\star(e) \end{cases}$$

Step 3: Optimal Policy For Folded POMDP

Key steps

1. Prove using **forward induction method** on the value iterates, $V_{n_{fold}}(e, b)$
2. Show (C) holds for $n+1$ given that (A)-(B) hold for n
3. Show (A)-(B) hold for $n+1$ given (C) holds for $n+1$ and (A)-(B) hold for n

Step 3: Optimal Policy For Folded POMDP

- $Q_{{n+1}_{fold}}(e, b; 0)$ is concave in b
- $Q_{{n+1}_{fold}}(e, b; 1)$ is linear in b
- $Q_{{n+1}_{fold}}(e, 0; 1) \geq Q_{{n+1}_{fold}}(e, 0; 0)$
- Case (i): $Q_{{n+1}_{fold}}(e, 1; 1) < Q_{{n+1}_{fold}}(e, 1; 0)$

Step 3: Optimal Policy For Folded POMDP

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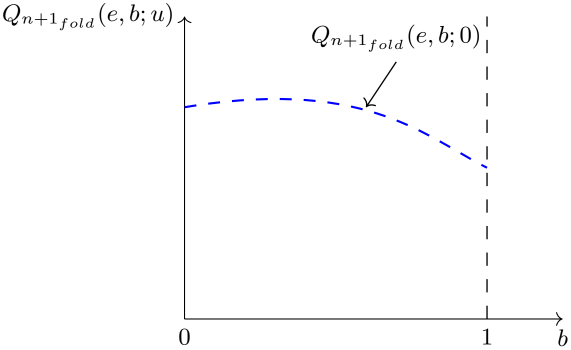

Step 3: Optimal Policy For Folded POMDP

- $Q_{n+1_{fold}}(e,b;0)$ is concave in b
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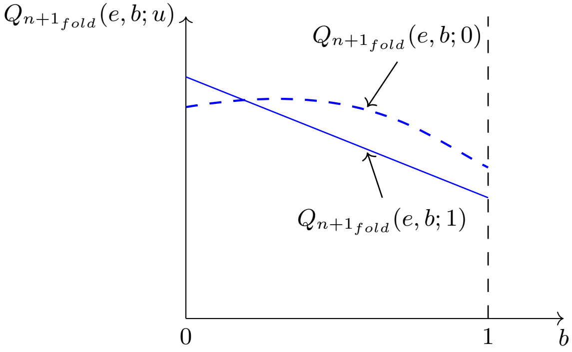

Step 3: Optimal Policy For Folded POMDP

- $Q_{{n+1}_{fold}}(e, b; 0)$ is concave in b
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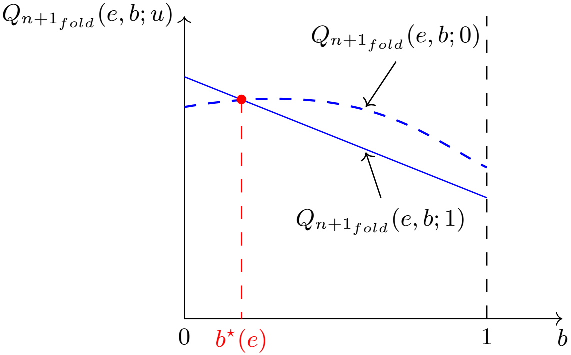
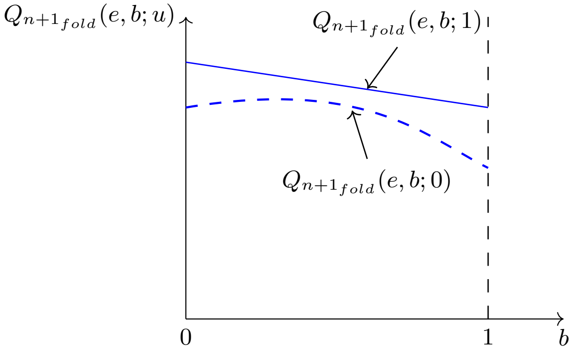


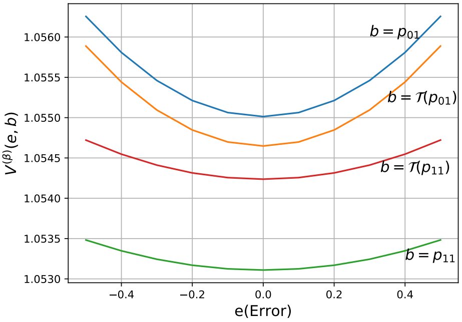
Step 3: Optimal Policy For Folded POMDP

- $Q_{n+1_{fold}}(e, b; 0)$ is concave in b
- $Q_{n+1_{fold}}(e, b; 1)$ is linear in b
- $Q_{n+1_{fold}}(e, 0; 1) \geq Q_{n+1_{fold}}(e, 0; 0)$
- Case (ii): $Q_{n+1_{fold}}(e, 1; 1) \geq Q_{n+1_{fold}}(e, 1; 0)$



Using *evenness* of original POMDP and *equivalence* of folded POMDP,

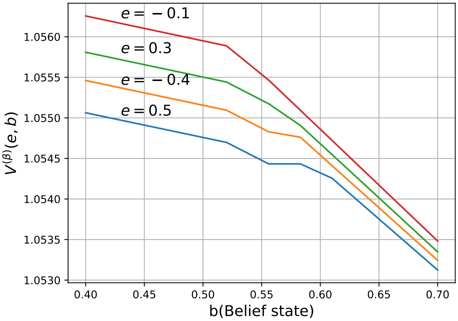
(A) For each b, $V^{(\beta)}(e,b)$ is non-decreasing in $|e|, e \in \mathbb{R}$



Using *evenness* of original POMDP and *equivalence* of folded POMDP,

(A) For each b, $V^{(\beta)}(e, b)$ is non-decreasing in $|e|, e \in \mathbb{R}$

(B) For each $e \in \mathbb{R}$, $V^{(\beta)}(e, b)$ is non-increasing in b



Step 4: Unfolding

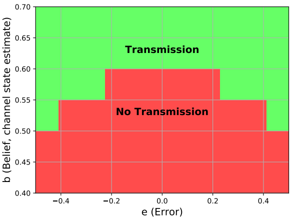
Using *evenness* of original POMDP and *equivalence* of folded POMDP,

(A) For each b, $V^{(\beta)}(e, b)$ is non-decreasing in $|e|, e \in \mathbb{R}$

(B) For each $e \in \mathbb{R}$, $V^{(\beta)}(e, b)$ is non-increasing in b

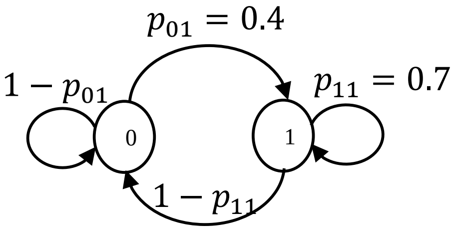
(C) For each $e \in \mathbb{R}$, there exits a threshold $b^\star(|e|)$ s.t.,

$$u = \begin{cases} 1 & \text{if } b \geq b^\star(|e|), \\ 0 & \text{if } b < b^\star(|e|) \end{cases}$$



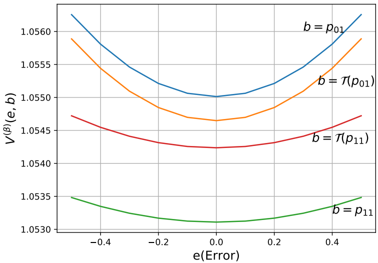
Set-up

- Discount factor $\beta = .99$
- Transmission price $\lambda = 0.65$ units
- AR process: $a = 0.7, w \sim \mathcal{N}(0,1)$
- Channel parameters:

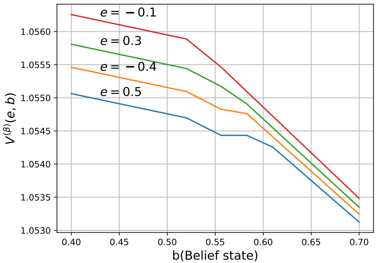


Gilbert-Elliot channel

Numerical Simulation



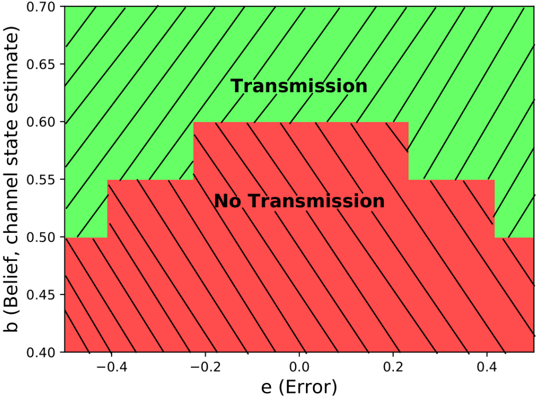
Change in $V^{(\beta)}$ with e



Change in $V^{(\beta)}$ with b

- $V^{(\beta)}(e,b)$ is even in e and non-decreasing in $|e|$
- $V^{(\beta)}(e,b)$ is non-increasing in b

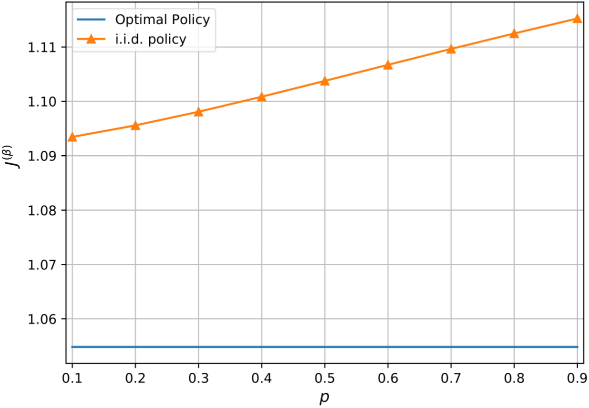
Numerical Simulation



Optimal policy

- $\phi^\star(e, b)$ is even in e
- $\phi^\star(e, b)$ exhibits a threshold structure w.r.t. b

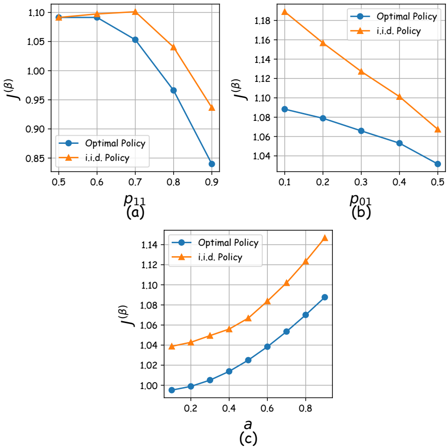
- Performance comparison with an i.i.d. policy with transmission probability p



Performance comparison as p is varied

Numerical simulation

- Performance comparison with an i.i.d. policy with transmission probability p =average energy consumption of optimal policy



Different system parameters: (a) $p_{01} = 0.4$ and $a = 0.7$, p_{11} varied; (b) p_{01} varied, $p_{11} = 0.7$ and $a = 0.7$; (c) $p_{01} = 0.4$ and $p_{11} = 0.7$ fixed, a varied

Extension: Joint optimality

Problem

$$\min_{\phi^{sen},\phi^{est}} \mathbb{E}_\phi \left(\sum_{t=0}^{\infty} \beta^t \left((x(t) - \hat{x}(t))^2 + \lambda u(t) \right) \right)$$

$\hat{x}(t) = \phi_t^{est}(\mathcal{F}^{est}(t)), u(t) = \phi_t^{sen}(\mathcal{F}^{sen}(t))$

Theorem 2: Joint optimality of sensor and estimator

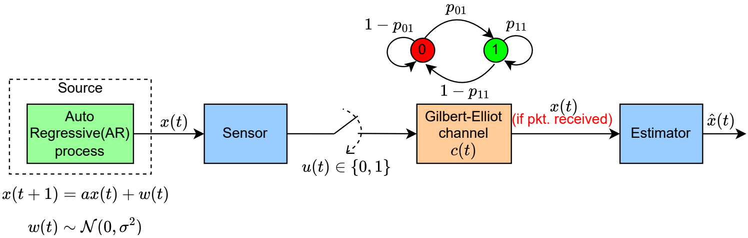
Estimator: $\hat{x}(t) = \begin{cases} x(t) & \text{if pkt. received,} \\ a\hat{x}(t-1) & \text{otherwise} \end{cases}$

Sensor: $u(t) = \begin{cases} 1 & \text{if } b(t) \geq b^\star(|e(t)|), \\ 0 & \text{if } b(t) < b^\star(|e(t)|) \end{cases}$

are jointly optimal

Optimal Risk-Sensitive Scheduling Policies for Remote Estimation of Autoregressive Markov Processes

(https://arxiv.org/abs/2403.13898v1)



Sensor Channel state observed by the sensor

Estimator $\hat{x}(t) = \begin{cases} x(t) & \text{if pkt. received,} \\ a\hat{x}(t-1) & \text{otherwise} \end{cases}$

Optimization problem

Problem

$$\min_{\phi} \ \mathbb{E}_{\phi}\left[exp\left(\gamma \sum_{t=0}^{T} (x(t) - \hat{x}(t))^2 + \lambda u(t) \right) \right],$$

risk-sensitivity parameter $\gamma > 0$, $\lambda u(t)$ is the communication cost $\lambda > 0, u(t) \in \{0, 1\}$

Advantages

- More general than risk-neutral optimization
- Penalize higher order moments of costs
- Robust to variations in system parameters

Contributions

Challenges

- Infinite horizon discounted MDP might not admit stationary policy
- Multiplicative in nature; policy depends on history

Challenges

- Infinite horizon discounted MDP might not admit stationary policy
- Multiplicative in nature; policy depends on history

Contributions

- Formulate finite horizon problem as MDP
- Show existence of optimal deterministic Markov policy
- Introduce "folded MDP" to ease analysis
- Establish the existence of a threshold-type optimal scheduling policy w.r.t. error

MDP Formulation

State **Error** $e(t) = x(t) - a\hat{x}(t-1)$

Channel state $c(t) \in \{0, 1\}$

Control $u(t) = \phi_t(\mathcal{F}(t)) \in \{0, 1\}$

where $\mathcal{F}(t)$ is the information available till time t

Instantaneous cost $d(e, c, u) := (1 - uc)e^2 + \lambda u$

Transmission strategy $\phi = \{\phi_t\}_{t=0}^{T}$

MDP

$$\min_{\phi} \; \mathbb{E}_{\phi} exp \left(\gamma \sum_{t=0}^{T} d(e(t), c(t), u(t)) \right)$$

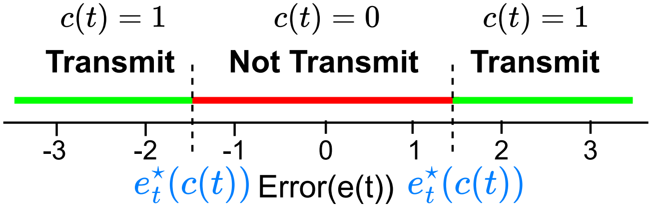
Main Result

Theorem 1: Structure of optimal transmission strategy

There exists a threshold-type optimal strategy,

$$u(t) = \phi^\star(e(t), c(t)) = \begin{cases} 1 & \text{if } |e(t)| \geq e^\star(c(t)), \\ 0 & \text{if } |e(t)| < e^\star(c(t)) \end{cases}$$

where $e^\star(c(t))$ is threshold



Proof Outline

Step 1 Value iteration to solve the MDP

Key challenge

- Multiplicative Bellman equation
- Instantaneous cost is unbounded
- State-space consists of error taking negative values

Step 2 Equivalent simpler folded MDP

Step 3 Optimal transmission strategy for folded MDP

Step 4 Unfolding

Step 1: Value Iteration

$$V(e,b) := \min_{\phi} \mathbb{E}_{\phi} exp\left(\gamma \sum_{t=0}^{T} d(e(t), c(t), u(t))\right)$$

Value iterates

- $V_0(e,c) = 0$
- $V_{t+1}(e,c) = \min_{u \in \{0,1\}} Q^{(\beta)}_{t+1}(e,c;u)$ where,

$$Q^{(\beta)}_{t+1}(e,c;u) = exp(\gamma d(e,c,u))\mathbb{E}_{e_+,c_+ \sim p(.,.|e,c;u)}[V_t(e_+,c_+)]$$

Step 1: Value Iteration

$$V(e,b) := \min_{\phi} \mathbb{E}_{\phi} exp\left(\gamma \sum_{t=0}^{T} d(e(t), c(t), u(t))\right)$$

Value iterates

- $V_0(e,c) = 0$
- $V_{t+1}(e,c) = \min_{u \in \{0,1\}} Q_{t+1}^{(\beta)}(e,c;u)$ where,

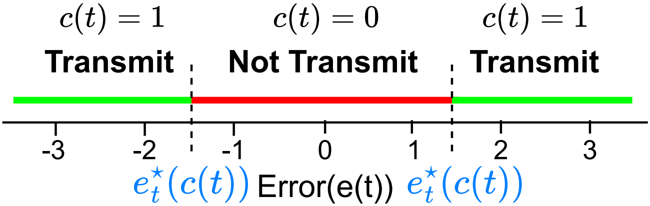
  $Q_{t+1}^{(\beta)}(e,c;u) = exp(\gamma d(e,c,u)) \mathbb{E}_{e_+,c_+ \sim p(.,.|e,c;u)}[V_t(e_+,c_+)]$
- $V_T(e,c) = V(e,c)$
- $\phi_t^{\star}(e,c) \in \operatorname{argmin}_{u \in \{0,1\}} Q_t(e,c;u)$ is optimal

Step 3: Optimal Policy for folded MDP

State-space folded MDP $\mathbb{R}_+ \times \{0,1\}$

Threshold policy (folded MDP)

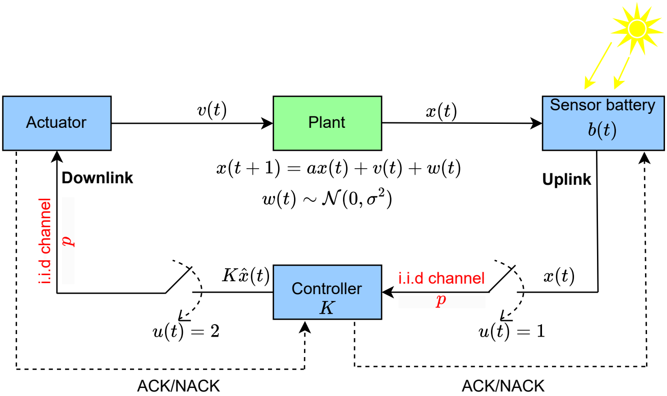
1. $c_t = 0$: $Q_t^{fold}(e,0;0) \leq Q_t^{fold}(e,0;1)$
2. $c_t = 1$: If $Q_t^{fold}(e,1;1) \leq Q_t^{fold}(e,1;0)$, then
$$Q_t^{fold}(e',1;1) \leq Q_t^{fold}(e',1;0) \text{ for all } e' \geq e$$



Optimal Scheduling of Uplink-Downlink Networked Control Systems with Energy Harvesting Sensor

(https://arxiv.org/abs/2403.14189v1)

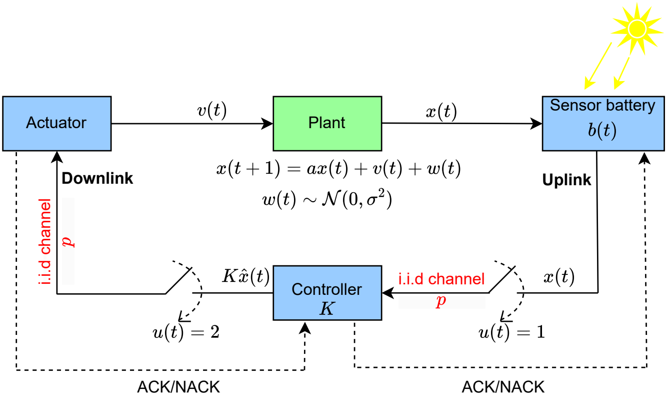
Half-duplex controller



Sensor Battery operated, $b(t) \in \{0, 1, \ldots, B\}$

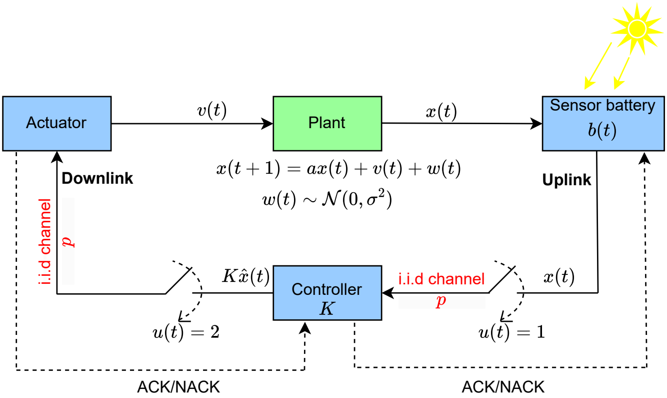
$u(t) \in \{0, 1, 2\}$

Half-duplex controller



$$\text{Estimator } \hat{x}(t+1) = \begin{cases} ax(t) & \text{if pkt. received,} \\ a\hat{x}(t) & \text{otherwise} \end{cases}$$

Half-duplex controller



Controller ***Half-duplex***: Activate either uplink or downlink channel

Trade-off between plant state estimation and timely plant control

Optimization Problem

Problem

$$\min_{\phi} \ \mathbb{E}_{\phi}\left(\sum_{t=0}^{\infty} \beta^t x(t)^2\right),$$

discount factor $\beta \in (0,1)$

Contributions

- Formulate infinite horizon problem as MDP
- Identify dynamic programming decomposition
- Show existence of a low complexity optimal scheduling policy

MDP formulation

State **Plant** $x(t) \in \mathbb{R}$

Age of packet at the controller $\tau(t) \in \mathbb{Z}_+$

Availability of control packet at the controller $y(t) \in \{0, 1\}$

Sensor battery energy level $b(t) \in \{0, 1, \dots, B\}$

MDP formulation

State **Plant** $x(t) \in \mathbb{R}$

Age of packet at the controller $\tau(t) \in \mathbb{Z}_+$

Availability of control packet at the controller $y(t) \in \{0, 1\}$

Sensor battery energy level $b(t) \in \{0, 1, \ldots, B\}$

Control $u(t) = \phi_t(\mathcal{F}(t)) \in \{0, 1, 2\}$

where $\mathcal{F}(t)$ is the information available till time t

Instantaneous cost $d(x, \tau, y, b, u) := x^2$

Scheduling strategy $\phi = \{\phi_t\}_{t=0}^{\infty}$

Technical Assumptions

Plant: $x(t+1) = ax(t) + v(t) + w(t)$

Controller Gain: K

One-step controllable

(A1) $a + K = 0$

Finiteness

(A2) There exists a ϕ such that

$$\mathbb{E}_\phi \left(\sum_{t=0}^{\infty} \beta^t x(t)^2 \right) < \infty$$

Main Result

Theorem 1: Structure of optimal scheduling strategy

There exists a threshold-type optimal strategy, i.e.,
1) if $u(t-1) = 0$, then

$$u(t) = \phi^\star(x(t), \tau(t), 1, b(t)) = 2 \quad \text{if } |x(t)| \geq x^\star(\tau(t), b(t))$$

2) if $u(t-1) = 1$, then

$$u(t) = \phi^\star(x(t), \tau(t), 1, b(t)) = 2 \quad \text{if } |x(t)| \geq x^\star(\tau(t), b(t))$$

where $x^\star(\tau(t), b(t))$ is threshold

Conclusion

- Minimize cumulative expected cost incurred
- Various assumptions on communication channel and system setup
- Posed as POMDP (MDP); analysis is hard
- Construct a simpler folded POMDP (MDP) equivalent to the original POMDP (MDP)
- Derive structural results of the optimal policy

Thank you!

- Chakravorty, Jhelum and Aditya Mahajan (2017). "Structure of optimal strategies for remote estimation over Gilbert-Elliott channel with feedback". In: *2017 IEEE International Symposium on Information Theory (ISIT).* IEEE, pp. 1272–1276.
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